EIGHTH INTERNATIONAL EDITION

Instructor's solution manual for

# Microelectronic Circuits

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## Chapter 1

### *Solutions to Exercises within the Chapter*

**Ex: 1.1** When output terminals are open-circuited, as in Fig. 1.1a:

For circuit a. $v_{oc} = v_s(t)$

For circuit b. $v_{oc} = i_s(t) \times R_s$

When output terminals are short-circuited, as in Fig. 1.1b:

For circuit a. $i_{sc} = \dfrac{v_s(t)}{R_s}$

For circuit b. $i_{sc} = i_s(t)$

For equivalency

$R_s i_s(t) = v_s(t)$

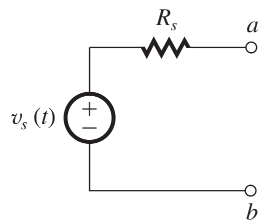



Figure 1.1a

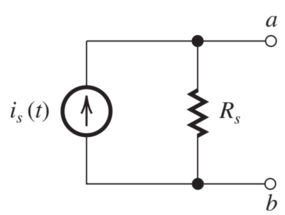
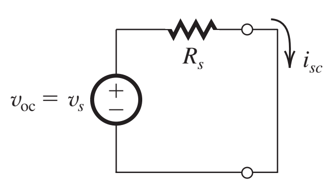



Figure 1.1b

**Ex: 1.2**



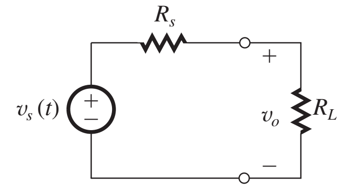

$v_{oc} = 10$ mV

$i_{sc} = 10\ \mu$A

$$R_s = \frac{v_{oc}}{i_{sc}} = \frac{10\ \text{mV}}{10\ \mu\text{A}} = 1\ \text{k}\Omega$$

**Ex: 1.3** Using voltage divider:

$$v_o(t) = v_s(t) \times \frac{R_L}{R_s + R_L}$$




Given $v_s(t) = 10$ mV and $R_s = 1$ kΩ.

If $R_L = 100$ kΩ

$$v_o = 10\ \text{mV} \times \frac{100}{100 + 1} = 9.9\ \text{mV}$$

If $R_L = 10$ kΩ

$$v_o = 10\ \text{mV} \times \frac{10}{10 + 1} \simeq 9.1\ \text{mV}$$

If $R_L = 1$ kΩ

$$v_o = 10\ \text{mV} \times \frac{1}{1 + 1} = 5\ \text{mV}$$

If $R_L = 100\ \Omega$

$$v_o = 10\ \text{mV} \times \frac{100}{100 + 1\ \text{K}} \simeq 0.91\ \text{mV}$$

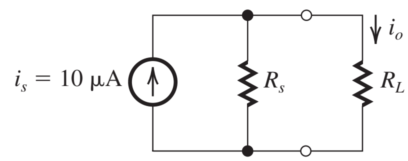
For $v_o = 0.8 v_s$,

$$\frac{R_L}{R_L + R_s} = 0.8$$

Since $R_s = 1$ kΩ,

$R_L = 4$ kΩ

**Ex: 1.4** Using current divider:




$$i_o = i_s \times \frac{R_s}{R_s + R_L}$$

Given $i_s = 10\ \mu$A, $R_s = 100$ kΩ.

For

$$R_L = 1\ \text{k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 1} = 9.9\ \mu\text{A}$$

For

$$R_L = 10\ \text{k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 10} \simeq 9.1\ \mu\text{A}$$

For

$$R_L = 100\ \text{k}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100}{100 + 100} = 5\ \mu\text{A}$$

$$\text{For } R_L = 1\ \text{M}\Omega,\ i_o = 10\ \mu\text{A} \times \frac{100\ \text{K}}{100\ \text{K} + 1\ \text{M}}$$

$\simeq 0.9\ \mu$A

$$\text{For } i_o = 0.8 i_s,\ \frac{100}{100 + R_L} = 0.8$$

$\Rightarrow R_L = 25$ kΩ

**Ex: 1.5** $f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000 \text{ Hz}$

$\omega = 2\pi f = 2\pi \times 10^3 \text{ rad/s}$

**Ex: 1.6** (a) $T = \frac{1}{f} = \frac{1}{60} \text{ s} = 16.7 \text{ ms}$

(b) $T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000 \text{ s}$

(c) $T = \frac{1}{f} = \frac{1}{10^6} \text{ s} = 1 \text{ μs}$

**Ex: 1.7** If 6 MHz is allocated for each channel, then 470 MHz to 608 MHz will accommodate

$$\frac{806 - 470}{6} = 23 \text{ channels}$$

Since the broadcast band starts with channel 14, it will go from channel 14 to channel 36.

**Ex: 1.8** $P = \frac{1}{T}\int_0^T \frac{v^2}{R}\, dt$

$$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$$

Alternatively,

$$P = P_1 + P_3 + P_5 + \cdots$$

$$= \left(\frac{4V}{\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{3\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{5\sqrt{2}\pi}\right)^2 \frac{1}{R} + \cdots$$

$$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots\right)$$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches $\pi^2/8$; thus $P$ becomes $V^2/R$ as found from direct calculation.

Fraction of energy in fundamental

$$= 8/\pi^2 = 0.81$$

Fraction of energy in first five harmonics

$$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.93$$

Fraction of energy in first seven harmonics

$$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right) = 0.95$$

Fraction of energy in first nine harmonics

$$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}\right) = 0.96$$

Note that 90% of the energy of the square wave is in the first three harmonics, that is, in the fundamental and the third harmonic.

**Ex: 1.9** (a) $D$ can represent 15 equally-spaced values between 0 and 3.75 V. Thus, the values are spaced 0.25 V apart.

$v_A = 0 \text{ V} \Rightarrow D = 0000$

$v_A = 0.25 \text{ V} \Rightarrow D = 0000$

$v_A = 1 \text{ V} \Rightarrow D = 0000$

$v_A = 3.75 \text{ V} \Rightarrow D = 0000$

(b) (i) 1 level spacing: $2^0 \times +0.25 = +0.25 \text{ V}$

(ii) 2 level spacings: $2^1 \times +0.25 = +0.5 \text{ V}$

(iii) 4 level spacings: $2^2 \times +0.25 = +1.0 \text{ V}$

(iv) 8 level spacings: $2^3 \times +0.25 = +2.0 \text{ V}$

(c) The closest discrete value represented by $D$ is +1.25 V; thus $D = 0101$. The error is -0.05 V, or $-0.05/1.3 \times 100 = -4\%$.

**Ex: 1.10** Voltage gain $= 20 \log 100 = 40 \text{ dB}$

Current gain $= 20 \log 1000 = 60 \text{ dB}$

Power gain $= 10 \log A_p = 10 \log (A_v A_i)$

$= 10 \log 10^5 = 50 \text{ dB}$

**Ex: 1.11** $P_{dc} = 15 \times 8 = 120 \text{ mW}$

$$P_L = \frac{(6/\sqrt{2})^2}{1} = 18 \text{ mW}$$

$P_{dissipated} = 120 - 18 = 102 \text{ mW}$

$$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$$

**Ex: 1.12** $v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5} \text{ V} = 10 \text{ μV}$

$$P_L = v_o^2/R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11} \text{ W}$$

With the buffer amplifier:

$$v_o = 1 \times \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

$$= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10 + 10} = 0.25 \text{ V}$$

$$P_L = \frac{v_o^2}{R_L} = \frac{0.25^2}{10} = 6.25 \text{ mW}$$

Voltage gain = $\frac{v_o}{v_s} = \frac{0.25 \text{ V}}{1 \text{ V}} = 0.25 \text{ V/V}$

$= -12 \text{ dB}$

Power gain $(A_p) \equiv \frac{P_L}{P_i}$

where $P_L$ = 6.25 mW and $P_i = v_i i_1$,

$v_i = 0.5$ V and

$$i_i = \frac{1 \text{ V}}{1 \text{ M}\Omega + 1 \text{ M}\Omega} = 0.5 \text{ μA}$$

Thus,

$P_i = 0.5 \times 0.5 = 0.25\ \mu\text{W}$

and

$$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$$

$10 \log A_p = 44$ dB

**Ex: 1.13** Open-circuit (no load) output voltage = $A_{vo} v_i$

Output voltage with load connected

$$= A_{vo} v_i \frac{R_L}{R_L + R_o}$$

$$0.8 = \frac{1}{R_o + 1} \Rightarrow R_o = 0.25\ \text{k}\Omega = 250\ \Omega$$

**Ex: 1.14** $A_{vo} = 40\ \text{dB} = 100\ \text{V/V}$

$$P_L = \frac{v_o^2}{R_L} = \left(A_{vo} v_i \frac{R_L}{R_L + R_o}\right)^2 \Big/ R_L$$

$$= v_i^2 \times \left(100 \times \frac{1}{1+1}\right)^2 \Big/ 1000 = 2.5\ v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10{,}000}$$

$$A_p \equiv \frac{P_L}{P_i} = \frac{2.5 v_i^2}{10^{-4} v_i^2} = 2.5 \times 10^4\ \text{W/W}$$

$10 \log A_p = 44$ dB

**Ex: 1.15** Without stage 3 (see figure)

$$\frac{v_L}{v_s} = \left(\frac{1\ \text{M}\Omega}{100\ \text{k}\Omega + 1\ \text{M}\Omega}\right)(10)\left(\frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 1\ \text{k}\Omega}\right) \times (100)\left(\frac{100}{100 + 1\ \text{k}\Omega}\right)$$

$$\frac{v_L}{v_s} = (0.909)(10)(0.9901)(100)(0.0909)$$

$= 81.8$ V/V

**Ex: 1.16** Refer the solution to Example 1.3 in the text.

$$\frac{v_{i1}}{v_s} = 0.909\ \text{V/V}$$

$v_{i1} = 0.909\ v_s = 0.909 \times 1 = 0.909$ mV

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 9.9 \times 0.909 = 9\ \text{V/V}$$

$v_{i2} = 9 \times v_S = 9 \times 1 = 9$ mV

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s} = 90.9 \times 9.9 \times 0.909$$

$= 818$ V/V

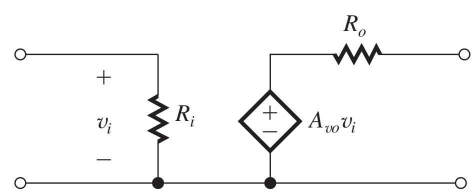
$v_{i3} = 818\ v_s = 818 \times 1 = 818$ mV

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744$ V/V

$v_L = 744 \times 1\ \text{mV} = 744$ mV

**Ex: 1.17** Using voltage amplifier model, the three-stage amplifier can be represented as




$R_i = 1\ \text{M}\Omega$

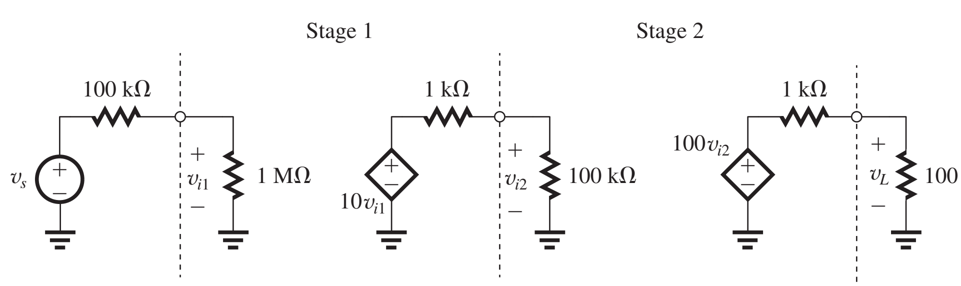
$R_o = 10\ \Omega$

$A_{vo} = A_{v1} \times A_{v2} \times A_{v3} = 9.9 \times 90.9 \times 1 = 900\ \text{V/V}$

The overall voltage gain

$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

This figure belongs to Exercise 1.15.



For $R_L = 10\ \Omega$:

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{10}{10 + 10} = 409\text{ V/V}$$

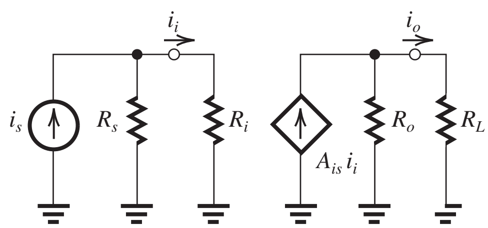
For $R_L = 1000\ \Omega$:

Overall voltage gain

$$= \frac{1\text{ M}}{1\text{ M} + 100\text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810\text{ V/V}$$

$\therefore$ Range of voltage gain is from 409 V/V to 810 V/V.

**Ex: 1.18**



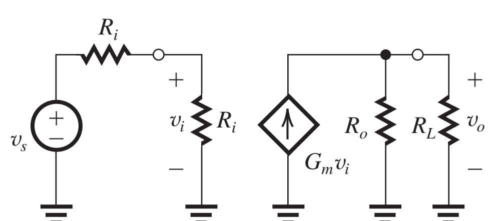

$$i_i = i_s \frac{R_s}{R_s + R_i}$$

$$i_o = A_{is} i_i \frac{R_o}{R_o + R_L} = A_{is} i_s \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

Thus,

$$\frac{i_o}{i_s} = A_{is} \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L}$$

**Ex: 1.19**




$$v_i = v_s \frac{R_i}{R_i + R_s}$$

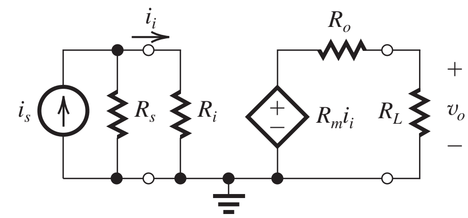
$$v_o = G_m v_i (R_o \parallel R_L)$$

$$= G_m v_s \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

Thus,

$$\frac{v_o}{v_s} = G_m \frac{R_i}{R_i + R_s} (R_o \parallel R_L)$$

**Ex: 1.20** Using the transresistance circuit model, the circuit will be




$$\frac{i_i}{i_s} = \frac{R_s}{R_i + R_s}$$

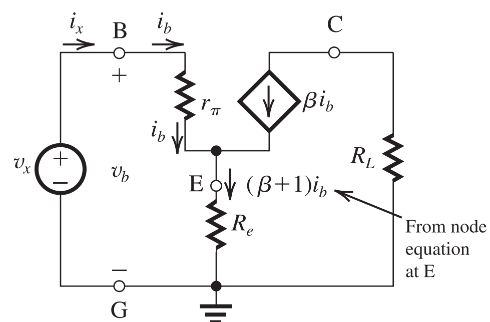
$$v_o = R_m i_i \times \frac{R_L}{R_L + R_o}$$

$$\frac{v_o}{i_i} = R_m \frac{R_L}{R_L + R_o}$$

Now $\dfrac{v_o}{i_s} = \dfrac{v_o}{i_i} \times \dfrac{i_i}{i_s} = R_m \dfrac{R_L}{R_L + R_o} \times \dfrac{R_s}{R_i + R_s}$

$$= R_m \frac{R_s}{R_s + R_i} \times \frac{R_L}{R_L + R_o}$$

**Ex: 1.21**




$$v_b = i_b r_\pi + (\beta + 1) i_b R_e$$

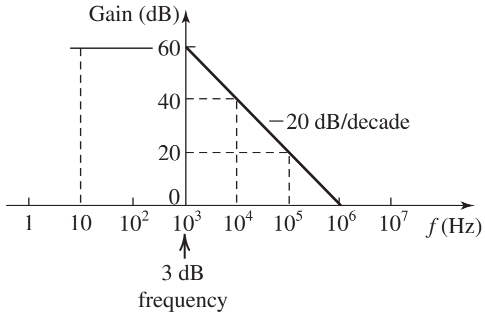
$$= i_b [r_\pi + (\beta + 1) R_e]$$

But $v_b = v_x$ and $i_b = i_x$, thus

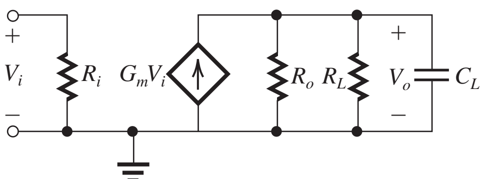
$$R_{\text{in}} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R_e$$

**Ex: 1.22**

| $f$ | Gain |
|---|---|
| 10 Hz | 60 dB |
| 10 kHz | 40 dB |
| 100 kHz | 20 dB |
| 1 MHz | 0 dB |




**Ex: 1.23**




$V_o = G_m V_i [R_o \parallel R_L \parallel C_L]$

$$= \frac{G_m V_i}{\dfrac{1}{R_o} + \dfrac{1}{R_L} + sC_L}$$

Thus, $\dfrac{V_o}{V_i} = \dfrac{G_m}{\dfrac{1}{R_o} + \dfrac{1}{R_L}} \times \dfrac{1}{1 + \dfrac{sC_L}{\dfrac{1}{R_o} + \dfrac{1}{R_L}}}$

$$\frac{V_o}{V_i} = \frac{G_m (R_L \parallel R_o)}{1 + sC_L (R_L \parallel R_o)}$$

which is of the STC LP type.

$$\omega_0 = \frac{1}{C_L (R_L \parallel R_o)}$$

$$= \frac{1}{4.5 \times 10^{-9} (10^3 \parallel R_o)}$$

For $\omega_0$ to be at least $w\pi \times 40 \times 10^3$, the highest value allowed for $R_o$ is

$$R_o = \frac{10^3}{2\pi \times 40 \times 10^3 \times 10^3 \times 4.5 \times 10^{-9} - 1}$$

$$= \frac{10^3}{1.131 - 1} = 7.64\ \text{k}\Omega$$

The dc gain is

$G_m (R_L \parallel R_o)$

To ensure a dc gain of at least 40 dB (i.e., 100), the minimum value of $G_m$ is

$\Rightarrow R_L \geq 100/(10^3 \parallel 7.64 \times 10^3) = 113.1$ mA/V

**Ex: 1.24** Refer to Fig. E1.24

$$\frac{V_2}{V_s} = \frac{R_i}{R_s + \dfrac{1}{sC} + R_i} = \frac{R_i}{R_s + R_i} \frac{s}{s + \dfrac{1}{C(R_s + R_i)}}$$

which is an HP STC function.

$$f_{3\text{dB}} = \frac{1}{2\pi C (R_s + R_i)} \leq 100\ \text{Hz}$$

$$C \geq \frac{1}{2\pi (1 + 9) 10^3 \times 100} = 0.16\ \mu\text{F}$$

**Ex: 1.25** $T = 50$ K

$n_i = BT^{3/2} e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15} (50)^{3/2}\ e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 50)}$

$\simeq 9.6 \times 10^{-39}/\text{cm}^3$

$T = 350$ K

$n_i = BT^{3/2} e^{-Eg/(2kT)}$

$= 7.3 \times 10^{15} (350)^{3/2}\ e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 350)}$

$= 4.15 \times 10^{11}/\text{cm}^3$

**Ex: 1.26** $N_D = 10^{17}/\text{cm}^3$

From Exercise 1.26, $n_i$ at

$T = 350\ \text{K} = 4.15 \times 10^{11}/\text{cm}^3$

$n_n = N_D = 10^{17}/\text{cm}^3$

$$p_n \cong \frac{ni^2}{N_D}$$

$$= \frac{(4.15 \times 10^{11})^2}{10^{17}}$$

$= 1.72 \times 10^6/\text{cm}^3$

**Ex: 1.27** At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

$p_p = N_A$

Want electron concentration

$$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4/\text{cm}^3$$

$$\therefore N_A = p_p = \frac{ni^2}{n_p}$$

$$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$$

$= 1.5 \times 10^{16}/\text{cm}^3$

**Ex: 1.28** (a) $v_{n-\text{drift}} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to $E$.

We use

$v_{n\text{-drift}} = 1350 \times \frac{1}{2 \times 10^{-4}} \because 1\ \mu\text{m} = 10^{-4}$ cm

$= 6.75 \times 10^6$ cm/s $= 6.75 \times 10^4$ m/s

(b) Time taken to cross 2-μm

length $= \frac{2 \times 10^{-6}}{6.75 \times 10^4} \simeq 30$ ps

(c) In $n$-type silicon, drift current density $J_n$ is

$J_n = qn\mu_n E$

$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1\text{ V}}{2 \times 10^{-4}}$

$= 1.08 \times 10^4$ A/cm$^2$

(d) Drift current $I_n = AJ_n$

$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$

$= 27\ \mu$A

The resistance of the bar is

$$R = \rho \times \frac{L}{A}$$

$$= qn\mu_n \times \frac{L}{A}$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{2 \times 10^{-4}}{0.25 \times 10^{-8}}$$

$$= 37.0\ \text{k}\Omega$$

Alternatively, we may simply use the preceding result for current and write

$R = V/I_n = 1$ V/27 μA $= 37.0$ kΩ

Note that $0.25\ \mu\text{m}^2 = 0.25 \times 10^{-8}\ \text{cm}^2$.

**Ex: 1.29** $J_n = qD_n \frac{dn(x)}{dx}$

From Fig. E1.29,

$n_0 = 10^{17}/\text{cm}^3 = 10^5/(\mu\text{m})^3$

$D_n = 35\ \text{cm}^2/\text{s} = 35 \times (10^4)^2\ (\mu\text{m})^2/\text{s}$

$= 35 \times 10^8\ (\mu\text{m})^2/\text{s}$

$\frac{dn}{dx} = \frac{10^5 - 0}{0.5} = 2 \times 10^5 \mu\text{m}^{-4}$

$J_n = qD_n \frac{dn(x)}{dx}$

$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 2 \times 10^5$

$= 112 \times 10^{-6}\ \text{A}/\mu\text{m}^2$

$= 112\ \mu\text{A}/\mu\text{m}^2$

For $I_n = 1\ \text{mA} = J_n \times A$

$\Rightarrow A = \frac{1\text{ mA}}{J_n} = \frac{10^3\ \mu\text{A}}{112\ \mu\text{A}/(\mu\text{m})^2} \simeq 9\ \mu\text{m}^2$

**Ex: 1.30** Using Eq. (1.44),

$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T = 1350 \times 25.9 \times 10^{-3}$

$\cong 35\ \text{cm}^2/\text{s}$

$D_p = \mu_p V_T = 480 \times 25.9 \times 10^{-3}$

$\cong 12.4\ \text{cm}^2/\text{s}$

**Ex: 1.31** Equation (1.49)

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_0}$$

$$W^2 = \frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_0$$

$$V_0 = \frac{1}{2}\left(\frac{q}{\epsilon_s}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)W^2$$

**Ex: 1.32** In a $p^+n$ diode $N_A \gg N_D$

Equation (1.49) $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$

We can neglect the term $\frac{1}{N_A}$ as compared to $\frac{1}{N_D}$, thus

$$W \simeq \sqrt{\frac{2\epsilon_s}{qN_D} \cdot V_0}$$

Equation (1.50) $x_n = W\frac{N_A}{N_A + N_D}$

$\simeq W\frac{N_A}{N_A}$

$= W$

Equation (1.51), $x_p = W\frac{N_D}{N_A + N_D}$

since $N_A \gg N_D$

$\simeq W\frac{N_D}{N_A} = W\Big/\left(\frac{N_A}{N_D}\right)$

Equation (1.52), $Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W$

$\simeq Aq\frac{N_A N_D}{N_A}W$

$= AqN_D W$

Equation (1.53), $Q_J = A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right)V_0}$

$\simeq A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A}\right)V_0}$ since $N_A \gg N_D$

$= A\sqrt{2\epsilon_s q N_D V_0}$

**Ex: 1.33** In Example 1.10, $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$

In the *n*-region of this *pn* junction

$n_n = N_D = 10^{16}/\text{cm}^3$

$p_n = \frac{n_i^2}{n_n} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4/\text{cm}^3$

As one can see from above equation, to increase minority-carrier concentration ($p_n$) by a factor of 2, one must lower $N_D$ (= $n_n$) by a factor of 2.

**Ex: 1.34**

Equation (1.64) $I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$

since $\frac{D_p}{L_p}$ and $\frac{D_n}{L_n}$ have approximately similar values, if $N_A \gg N_D$, then the term $\frac{D_n}{L_n N_A}$ can be neglected as compared to $\frac{D_p}{L_p N_D}$

$\therefore I_S \cong Aqn_i^2\frac{D_p}{L_p N_D}$

**Ex: 1.35** $I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$

$= 10^{-4}\times1.6\times10^{-19}\times(1.5\times10^{10})^2$

$\times\left(\frac{10}{5\times10^{-4}\times\frac{10^{16}}{2}} + \frac{18}{10\times10^{-4}\times10^{18}}\right)$

$= 1.46\times10^{-14}$ A

$I = I_S(e^{V/V_T} - 1)$

$\simeq I_S e^{V/V_T} = 1.45\times10^{-14}e^{0.605/(25.9\times10^{-3})}$

$= 0.2$ mA

**Ex: 1.36** $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 - V_F)}$

$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.814 - 0.605)}$

$= 1.66\times10^{-5}$ cm $= 0.166$ µm

**Ex: 1.37** $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$

$= \sqrt{\frac{2\times1.04\times10^{-12}}{1.6\times10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.814 + 2)}$

$= 6.08\times10^{-5}$ cm $= 0.608$ µm

Using Eq. (1.52),

$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W$

$= 10^{-4}\times1.6\times10^{-19}\left(\frac{10^{18}\times10^{16}}{10^{18}+10^{16}}\right)\times6.08\times10^{-5}$ cm

$= 9.63$ pC

Reverse current $I = I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$

$= 10^{-14}\times1.6\times10^{-19}\times(1.5\times10^{10})^2$

$\times\left(\frac{10}{5\times10^{-4}\times10^{16}} + \frac{18}{10\times10^{-4}\times10^{18}}\right)$

$= 7.3\times10^{-15}$ A

**Ex: 1.38** Equation (1.69),

$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$

$= 10^{-4}\sqrt{\left(\frac{1.04\times10^{-12}\times1.6\times10^{-19}}{2}\right)}$

$\sqrt{\left(\frac{10^{18}\times10^{16}}{10^{18}+10^{16}}\right)\left(\frac{1}{0.814}\right)}$

$= 3.2$ pF

Equation (1.70),

$C_j = \frac{C_{j0}}{\sqrt{1+\frac{V_R}{V_0}}}$

$= \frac{3.2\times10^{-12}}{\sqrt{1+\frac{2}{0.814}}}$

$= 1.72$ pF

**Ex: 1.39** $C_d = \frac{dQ}{dV} = \frac{d}{dV}(\tau_T I)$

$= \frac{d}{dV}[\tau_T\times I_S(e^{V/V_T} - 1)]$

$= \tau_T I_S\frac{d}{dV}(e^{V/V_T} - 1)$

$= \tau_T I_S\frac{1}{V_T}e^{V/V_T}$

$= \frac{\tau_T}{V_T}\times I_S e^{V/V_T}$

$\cong \left(\frac{\tau_T}{V_T}\right)I$

**Ex: 1.40** Equation (1.73),

$$\tau_p = \frac{L_p^2}{D_p}$$

$$= \frac{(5 \times 10^{-4})^2}{10}$$

$= 25$ ns

Equation (3.57),

$$C_d = \left(\frac{\tau_T}{V_T}\right)I$$

In Example 1.6, $N_A = 10^{18}/\text{cm}^3$,

$N_D = 10^{16}/\text{cm}^3$

Assuming $N_A \gg N_D$,

$\tau_T \simeq \tau_p = 25$ ns

$$\therefore C_d = \left(\frac{25 \times 10^{-9}}{25.9 \times 10^{-3}}\right) 0.1 \times 10^{-3}$$

$= 96.5$ pF

***Solutions to End-of-Chapter Problems***

**1.1** (a) $V = IR = 5\text{ mA} \times 1\text{ k}\Omega = 5\text{ V}$

$P = I^2R = (5\text{ mA})^2 \times 1\text{ k}\Omega = 25\text{ mW}$

(b) $R = V/I = 5\text{ V}/1\text{ mA} = 5\text{ k}\Omega$

$P = VI = 5\text{ V} \times 1\text{ mA} = 5\text{ mW}$

(c) $I = P/V = 100\text{ mW}/10\text{ V} = 10\text{ mA}$

$R = V/I = 10\text{ V}/10\text{ mA} = 1\text{ k}\Omega$

(d) $V = P/I = 1\text{ mW}/0.1\text{ mA}$

$= 10\text{ V}$

$R = V/I = 10\text{ V}/0.1\text{ mA} = 100\text{ k}\Omega$

(e) $P = I^2R \Rightarrow I = \sqrt{P/R}$

$I = \sqrt{1000\text{ mW}/1\text{ k}\Omega} = 31.6\text{ mA}$

$V = IR = 31.6\text{ mA} \times 1\text{ k}\Omega = 31.6\text{ V}$

*Note:* V, mA, kΩ, and mW constitute a consistent set of units.

**1.2** (a) $I = \dfrac{V}{R} = \dfrac{5\text{ V}}{1\text{ k}\Omega} = 5\text{ mA}$

(b) $R = \dfrac{V}{I} = \dfrac{5\text{ V}}{1\text{ mA}} = 5\text{ k}\Omega$

(c) $V = IR = 0.1\text{ mA} \times 10\text{ k}\Omega = 1\text{ V}$

(d) $I = \dfrac{V}{R} = \dfrac{1\text{ V}}{100\ \Omega} = 0.01\text{ A} = 10\text{ mA}$

*Note:* Volts, milliamps, and kilohms constitute a consistent set of units.

**1.3** (a) $P = I^2R = (20 \times 10^{-3})^2 \times 1 \times 10^3$

$= 0.4\text{ W}$

Thus, $R$ should have a $\frac{1}{2}$-W rating.

(b) $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3$

$= 1.6\text{ W}$

Thus, the resistor should have a 2-W rating.

(c) $P = I^2R = (1 \times 10^{-3})^2 \times 100 \times 10^3$

$= 0.1\text{ W}$

Thus, the resistor should have a $\frac{1}{8}$-W rating.

(d) $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3$

$= 0.16\text{ W}$

Thus, the resistor should have a $\frac{1}{4}$-W rating.

(e) $P = V^2/R = 20^2/(1 \times 10^3) = 0.4\text{ W}$

Thus, the resistor should have a $\frac{1}{2}$-W rating.

(f) $P = V^2/R = 11^2/(1 \times 10^3) = 0.121\text{ W}$

Thus, a rating of $\frac{1}{8}$ W should theoretically suffice, though $\frac{1}{4}$ W would be prudent to allow for inevitable tolerances and measurement errors.

**1.4** See figure on next page, which shows how to realize the required resistance values.

**1.5** Shunting the 10 kΩ by a resistor of value of $R$ results in the combination having a resistance $R_{eq}$,

$$R_{eq} = \frac{10R}{R+10}$$

Thus, for a 1% reduction,

$$\frac{R}{R+10} = 0.99 \Rightarrow R = 990\text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R+10} = 0.95 \Rightarrow R = 190\text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R+10} = 0.90 \Rightarrow R = 90\text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R+10} = 0.50 \Rightarrow R = 10\text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ results in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9\text{ k}\Omega$$

a 1% reduction;

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09\text{ k}\Omega$$

a 9.1% reduction;

(c) 10 kΩ results in

$$R_{eq} = \frac{10}{10 + 10} = 5\text{ k}\Omega$$

a 50% reduction.

**1.6** Use voltage divider to find $V_O$

$$V_O = 5\frac{2}{2+3} = 2\text{ V}$$

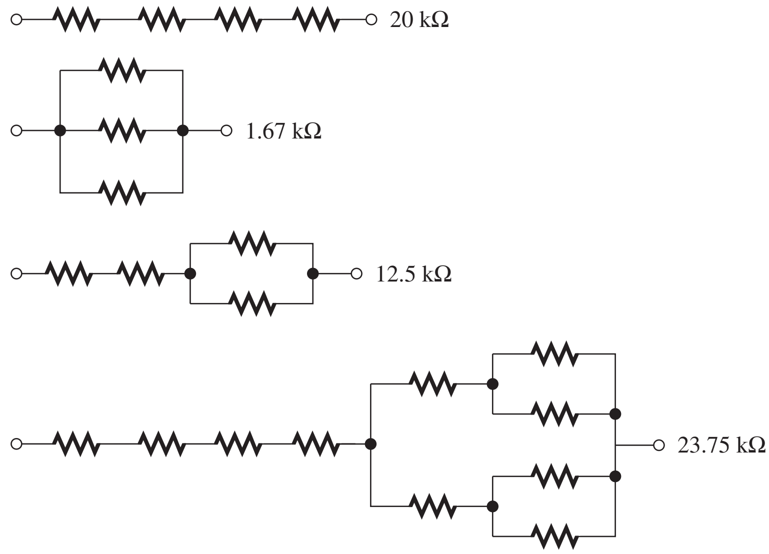
Equivalent output resistance $R_O$ is

$$R_O = (2\text{ k}\Omega \parallel 3\text{ k}\Omega) = 1.2\text{ k}\Omega$$

This figure belongs to Problem **1.4**.

All resistors are 5 kΩ



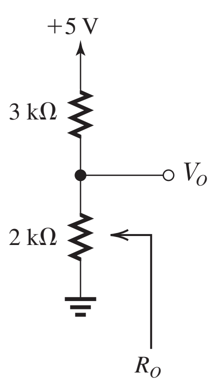

The extreme values of $V_O$ for ±5% tolerance resistor are

$$V_{O\min} = 5\frac{2(1-0.05)}{2(1-0.05)+3(1+0.05)}$$

$= 1.88$ V

$$V_{O\max} = 5\frac{2(1+0.05)}{2(1+0.05)+3(1-0.05)}$$

$= 2.12$ V



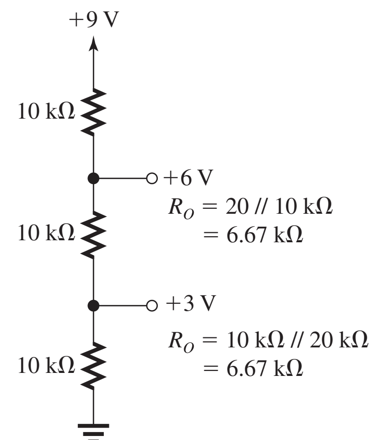

The extreme values of $R_O$ for ±5% tolerance resistors are $1.2 \times 1.05 = 1.26$ kΩ and $1.2 \times 0.95 = 1.14$ kΩ.

**1.7** $V_O = V_{DD}\dfrac{R_2}{R_1+R_2}$

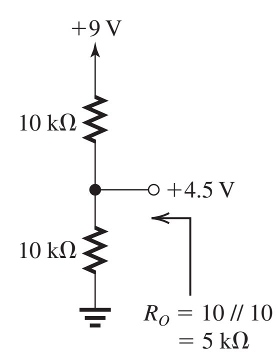
To find $R_O$, we short-circuit $V_{DD}$ and look back into node X,

$$R_O = R_2 \parallel R_1 = \frac{R_1R_2}{R_1+R_2}$$

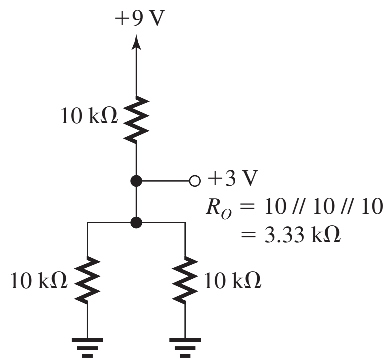
**1.8**



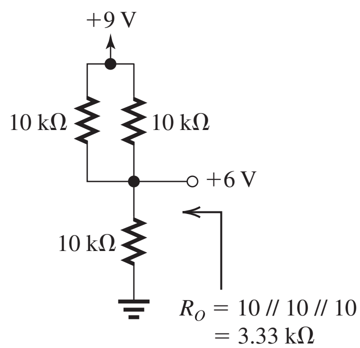

(a)




(b)




(c)




(d)

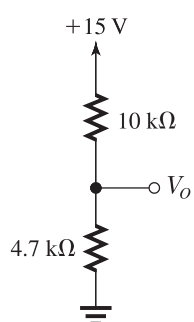
Voltage generated:

+3V [two ways: (a) and (c) with (c) having lower output resistance]

+4.5 V (b)

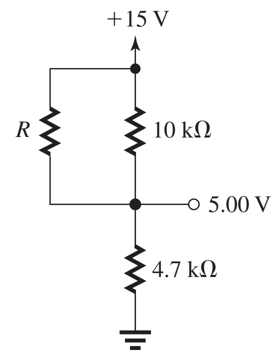
+6V [two ways: (a) and (d) with (d) having a lower output resistance]

**1.9**




$$V_O = 15\frac{4.7}{10 + 4.7} = 4.80 \text{ V}$$

To increase $V_O$ to 10.00 V, we shunt the 10-kΩ resistor by a resistor $R$ whose value is such that $10 \parallel R = 2 \times 4.7$.




Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

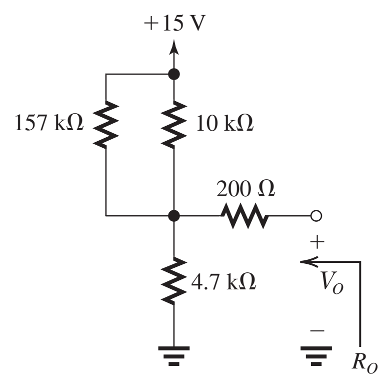
$$\Rightarrow R = 156.7 \approx 157 \text{ k}\Omega$$

Now,

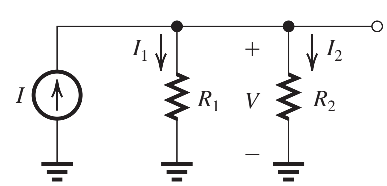
$$R_O = 10 \text{ k}\Omega \parallel R \parallel 4.7 \text{ k}\Omega$$

$$= 9.4 \parallel 4.7 = \frac{9.4}{3} = 3.133 \text{ k}\Omega$$

To make $R_O = 3.33$, we add a series resistance of approximately 200 Ω, as shown below,




**1.10**




$$V = I(R_1 \parallel R_2)$$

$$= I\frac{R_1 R_2}{R_1 + R_2}$$

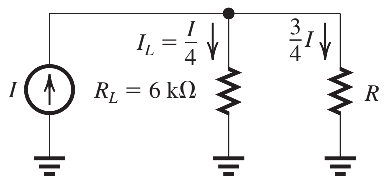
$$I_1 = \frac{V}{R_1} = I\frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$$

**1.11** Connect a resistor $R$ in parallel with $R_L$. To make $I_L = I/4$ (and thus the current through $R$, $3I/4$), $R$ should be such that

$$6I/4 = 3IR/4$$

$$\Rightarrow R = 2\ \text{k}\Omega$$




**1.12** The parallel combination of the resistors is $R_\parallel$ where

$$\frac{1}{R_\parallel} = \sum_{i=1}^{N} 1/R_i$$

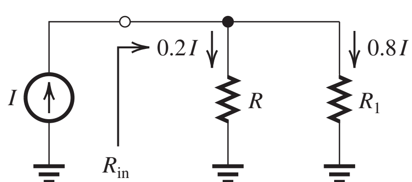
The voltage across them is

$$V = I \times R_\parallel = \frac{I}{\sum_{i=1}^{N} 1/R_i}$$

Thus, the current in resistor $R_k$ is

$$I_k = V/R_k = \frac{I/R_k}{\sum_{i=1}^{N} 1/R_i}$$

**1.13**




To make the current through $R$ equal to $0.2I$, we shunt $R$ by a resistance $R_1$ having a value such that the current through it will be $0.8I$; thus

$$0.2IR = 0.8IR_1 \Rightarrow R_1 = \frac{R}{4}$$

The input resistance of the divider, $R_{\text{in}}$, is

$$R_{\text{in}} = R \parallel R_1 = R \parallel \frac{R}{4} = \frac{1}{5}R$$

Now if $R_1$ is 10% too high, that is, if

$$R_1 = 1.1\frac{R}{4}$$

the problem can be solved in two ways:

(a) Connect a resistor $R_2$ across $R_1$ of value such that $R_2 \parallel R_1 = R/4$, thus

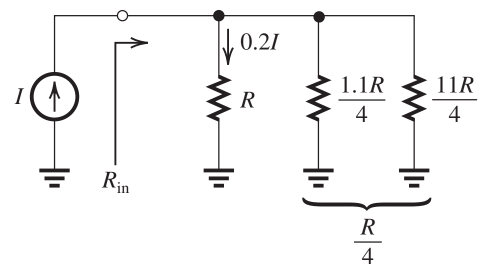
$$\frac{R_2(1.1R/4)}{R_2 + (1.1R/4)} = \frac{R}{4}$$

$$1.1R_2 = R_2 + \frac{1.1R}{4}$$

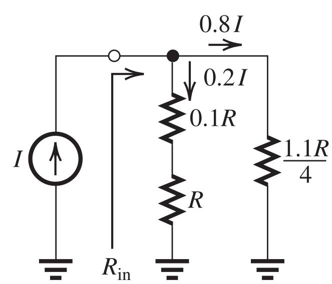
$$\Rightarrow R_2 = \frac{11R}{4} = 2.75\ \text{R}$$

$$R_{\text{in}} = R \parallel \frac{1.1R}{4} \parallel \frac{11R}{4}$$

$$= R \parallel \frac{R}{4} = \frac{R}{5}$$




(b) Connect a resistor in series with the load resistor $R$ so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of $R$ (i.e., $0.1R$).




$$R_{\text{in}} = 1.1R \parallel \frac{1.1R}{4}$$

$$= \frac{1.1R}{5}$$

that is, 10% higher than in case (a).

**1.14** For $R_L = 10\ \text{k}\Omega$ , when signal source generates $0-0.5$ mA, a voltage of $0-2$ V may appear across the source

To limit $v_s \leq 1$ V, the net resistance has to be $\leq 2\ \text{k}\Omega$. To achieve this we have to shunt $R_L$ with a resistor R so that $(R \parallel R_L) \leq 2\ \text{k}\Omega$.

$$R \parallel R_L \leq 2\ \text{k}\Omega.$$

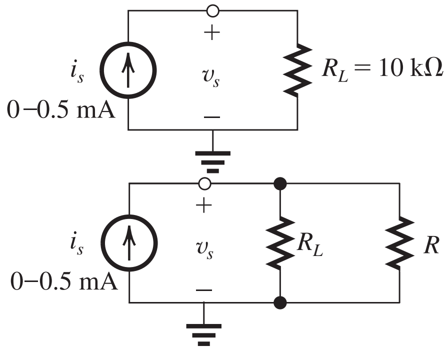
$$\frac{RR_L}{R + R_L} \leq 2\ \text{k}\Omega$$

For $R_L = 10\ \text{k}\Omega$

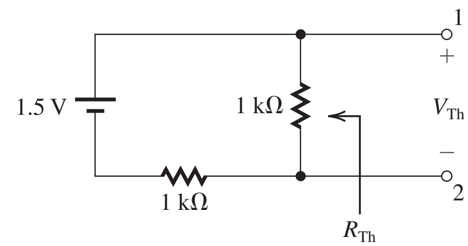
$$R \leq 2.5\ \text{k}\Omega$$

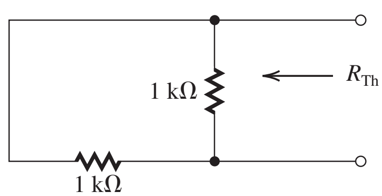
The resulting circuit needs only one additional resistance of $2\ \text{k}\Omega$ in parallel with $R_L$ so that

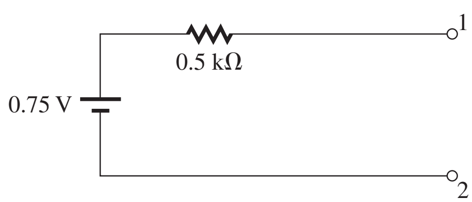
$v_s \leq 1$ V. The circuit is a current divider, and the current through $R_L$ is now 0–0.1 mA.



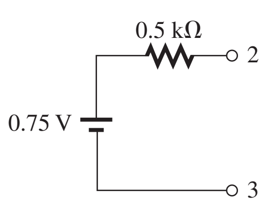

**1.15** (a) Between terminals 1 and 2:



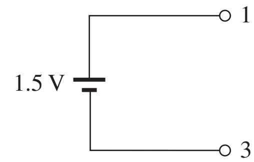



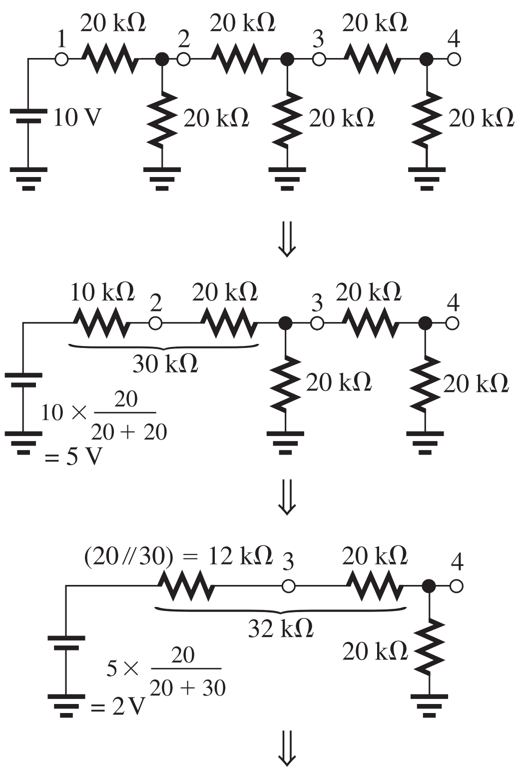

(b) Same procedure is used for (b) to obtain



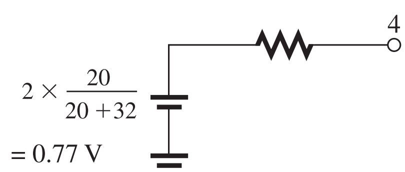

(c) Between terminals 1 and 3, the open-circuit voltage is 1.5 V. When we short circuit the voltage source, we see that the Thévenin resistance will be zero. The equivalent circuit is then

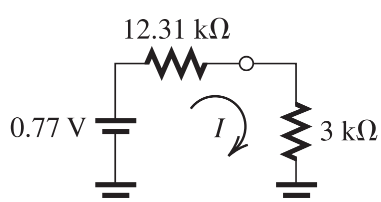



**1.16**




Thévenin equivalent: (20 // 32) = 12.31 kΩ



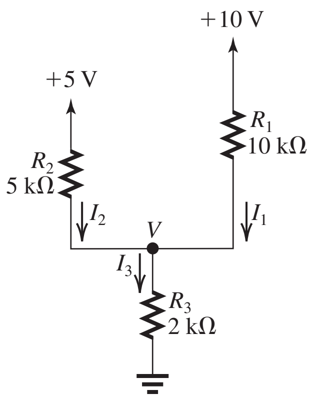

Now, when a resistance of 3 kΩ is connected between node 4 and ground,

$$I = \frac{0.77}{12.31 + 3}$$

$= 0.05$ mA

**1.17**




(a) Node equation at the common mode yields

$I_3 = I_1 + I_2$

Using the fact that the sum of the voltage drops across $R_1$ and $R_3$ equals 10 V, we write

$10 = I_1R_1 + I_3R_3$

$= 10I_1 + (I_1 + I_2) \times 2$

$= 12I_1 + 2I_2$

That is,

$12I_1 + 2I_2 = 10$ (1)

Similarly, the voltage drops across $R_2$ and $R_3$ add up to 5 V, thus

$5 = I_2R_2 + I_3R_3$

$= 5I_2 + (I_1 + I_2) \times 2$

which yields

$2I_1 + 7I_2 = 5$ (2)

Equations (1) and (2) can be solved together by multiplying Eq. (2) by 6:

$12I_1 + 42I_2 = 30$ (3)

Now, subtracting Eq. (1) from Eq. (3) yields

$40I_2 = 20$

$\Rightarrow I_2 = 0.5$ mA

Substituting in Eq. (2) gives

$2I_1 = 5 - 7 \times 0.5$ mA

$\Rightarrow I_1 = 0.75$ mA

$I_3 = I_1 + I_2$

$= 0.75 + 0.5$

$= 1.25$ mA

$V = I_3R_3$

$= 1.25 \times 2 = 2.5$ V

To summarize:

$I_1 = 0.75$ mA $\quad I_2 = 0.5$ mA

$I_3 = 1.25$ mA $\quad V = 2.5$ V

(b) A node equation at the common node can be written in terms of $V$ as

$$\frac{10 - V}{R_1} + \frac{5 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{10 - V}{10} + \frac{5 - V}{5} = \frac{V}{2}$$

$\Rightarrow 0.8V = 2$

$\Rightarrow V = 2.5$ V

Now, $I_1$, $I_2$, and $I_3$ can be easily found as

$$I_1 = \frac{10 - V}{10} = \frac{10 - 2.5}{10}$$

$= 0.75$ mA

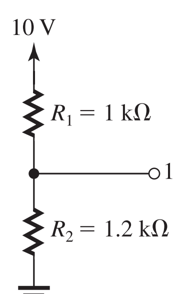
$$I_2 = \frac{5 - V}{5} = \frac{5 - 2.5}{5}$$

$= 0.5$ mA

$$I_3 = \frac{V}{R_3} = \frac{2.5}{2} = 1.25 \text{ mA}$$

Method (b) is much preferred, being faster, more insightful, and less prone to errors. In general, one attempts to identify the lowest possible number of variables and write the corresponding minimum number of equations.

**1.18** Find the Thévenin equivalent of the circuit to the left of node 1.

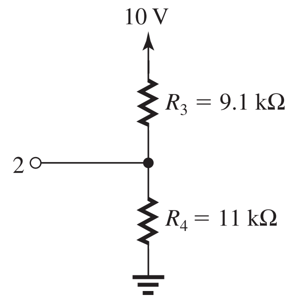



Between node 1 and ground,

$R_{Th} = (1\ \text{k}\Omega \parallel 1.2\ \text{k}\Omega) = 0.545\ \text{k}\Omega$

$$V_{Th} = 10 \times \frac{1.2}{1 + 1.2} = 5.45 \text{ V}$$

Find the Thévenin equivalent of the circuit to the right of node 2.

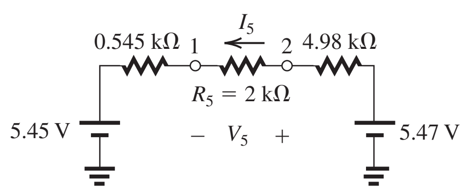



Between node 2 and ground,

$$R_{Th} = 9.1 \text{ k}\Omega \parallel 11 \text{ k}\Omega = 4.98 \text{ k}\Omega$$

$$V_{Th} = 10 \times \frac{11}{11 + 9.1} = 5.47 \text{ V}$$

The resulting simplified circuit is




$$I_5 = \frac{5.47 - 5.45}{4.98 + 2 + 0.545}$$

$$= 2.66 \ \mu\text{A}$$

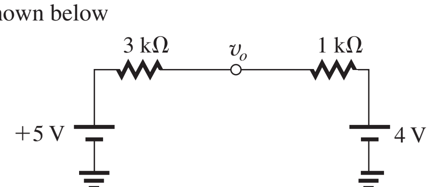
$$V_5 = 2.66 \ \mu\text{A} \times 2 \text{ k}\Omega$$

$$= 5.32 \text{ mV}$$

**1.19** We first find the Thévenin equivalent of the source to the right of $v_O$.

$$V = 4 \times 1 = 4 \text{ V}$$

Then, we may redraw the circuit in Fig. P1.19 as shown below




Then, the voltage at $v_O$ is found from a simple voltage division.

$$v_O = 4 + (5 - 4) \times \frac{1}{3 + 1} = 4.25 \text{ V}$$

**1.20** Refer to Fig. P1.20. Using the voltage divider rule at the input side, we obtain

$$\frac{v_\pi}{v_s} = \frac{r_\pi}{r_\pi + R_s} \qquad (1)$$

At the output side, we find $v_o$ by multiplying the current $g_m v_\pi$ by the parallel equivalent of $r_o$ and $R_L$,

$$v_o = -g_m v_\pi (r_o \parallel R_L) \qquad (2)$$

Finally, $v_o/v_s$ can be obtained by combining Eqs. (1) and (2) as

$$\frac{v_o}{v_s} = -\frac{r_\pi}{r_\pi + R_s} g_m (r_o \parallel R_L)$$

**1.21** (a) $T = 10^{-4}$ ms $= 10^{-7}$ s

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^7 \text{ rad/s}$$

(b) $f = 1$ GHz $= 10^9$ Hz

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

(c) $\omega = 6.28 \times 10^2$ rad/s

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

(d) $T = 10$ s

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

(e) $f = 60$ Hz

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$

$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

(f) $\omega = 1$ krad/s $= 10^3$ rad/s

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

(g) $f = 1900$ MHz $= 1.9 \times 10^9$ Hz

$$T = \frac{1}{f} = 5.26 \times 10^{-10} \text{ s}$$

$$\omega = 2\pi f = 1.194 \times 10^{10} \text{ rad/s}$$

**1.22** (a) $Z = R + \dfrac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$= (1 - j1.59)\ \text{k}\Omega$

(b) $Y = \dfrac{1}{R} + j\omega C$

$= \dfrac{1}{10^4} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$

$= 10^{-4}(1 + j6.28)\ \Omega$

$Z = \dfrac{1}{Y} = \dfrac{10^4}{1 + j6.28}$

$= \dfrac{10^4(1 - j6.28)}{1 + 6.28^2}$

$= (247.3 - j1553)\ \Omega$

(c) $Y = \dfrac{1}{R} + j\omega C$

$= \dfrac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$

$= 10^{-5}(1 + j0.628)$

$Z = \dfrac{10^5}{1 + j0.628}$

$= (71.72 - j45.04)\ \text{k}\Omega$

(d) $Z = R + j\omega L$

$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$

$= 100 + j6.28 \times 100$

$= (100 + j628),\ \Omega$

**1.23** (a) $Z = 1\ \text{k}\Omega$ at all frequencies

(b) $Z = 1/\text{j}\omega C = -j\dfrac{1}{2\pi f \times 10 \times 10^{-9}}$

At $f = 60$ Hz, $Z = -j265\ \text{k}\Omega$

At $f = 100$ kHz, $Z = -j159\ \Omega$

At $f = 1$ GHz, $Z = -j0.016\ \Omega$

(c) $Z = 1/\text{j}\omega C = -j\dfrac{1}{2\pi f \times 10 \times 10^{-12}}$

At $f = 60$ Hz, $Z = -j0.265\ \text{G}\Omega$

At $f = 100$ kHz, $Z = -j0.16\ \text{M}\Omega$

At $f = 1$ GHz, $Z = -j15.9\ \Omega$

(d) $Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$

At $f = 60$ Hz, $Z = j3.77\ \Omega$

At $f = 100$ kHz, $Z = j6.28\ \text{k}\Omega$

At $f = 1$ GHz, $Z = j62.8\ \text{M}\Omega$

(e) $Z = j\omega L = j2\pi f L = j2\pi f(1 \times 10^{-6})$

$f = 60$ Hz, $Z = j0.377\ \text{m}\Omega$

$f = 100$ kHz, $Z = j0.628\ \Omega$

$f = 1$ GHz, $Z = j6.28\ \text{k}\Omega$

**1.24** $Y = \dfrac{1}{j\omega L} + j\omega C$

$= \dfrac{1 - \omega^2 LC}{j\omega L}$

$\Rightarrow Z = \dfrac{1}{Y} = \dfrac{j\omega L}{1 - \omega^2 LC}$

The frequency at which $|Z| = \infty$ is found letting the denominator equal zero:

$1 - \omega^2 LC = 0$

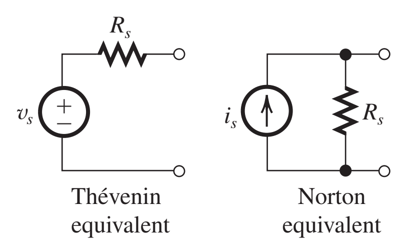
$\Rightarrow \omega = \dfrac{1}{\sqrt{LC}}$

At frequencies just below this, $\angle Z = +90°$.

At frequencies just above this, $\angle Z = -90°$.

Since the impedance is infinite at this frequency, the current drawn from an ideal voltage source is zero.

**1.25**




$v_{oc} = v_s$

$i_{sc} = i_s$

$v_s = i_s R_s$

Thus,

$R_s = \dfrac{v_{oc}}{i_{sc}}$

(a) $v_s = v_{oc} = 1$ V

$i_s = i_{sc} = 0.1$ mA

$R_s = \dfrac{v_{oc}}{i_{sc}} = \dfrac{1\ \text{V}}{0.1\ \text{mA}} = 10\ \text{k}\Omega$

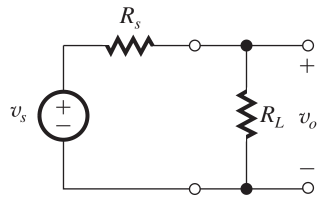
(b) $v_s = v_{oc} = 0.1$ V

$i_s = i_{sc} = 1\ \mu\text{A}$

$R_s = \dfrac{v_{oc}}{i_{sc}} = \dfrac{0.1\ \text{V}}{1\ \mu\text{A}} = 0.1\ \text{M}\Omega = 100\ \text{k}\Omega$

**1.26**




$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s}$$

$$v_o = v_s \bigg/ \left(1 + \frac{R_s}{R_L}\right)$$

Thus,

$$\frac{v_s}{1 + \frac{R_s}{100}} = 40 \qquad (1)$$

and

$$\frac{v_s}{1 + \frac{R_s}{10}} = 10 \qquad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$\frac{1 + (R_s/10)}{1 + (R_s/100)} = 4$$

$$\Rightarrow R_S = 50 \text{ k}\Omega$$

Substituting in Eq. (2) gives

$$v_s = 60 \text{ mV}$$

The Norton current $i_s$ can be found as

$$i_s = \frac{v_s}{R_s} = \frac{60 \text{ mV}}{50 \text{ k}\Omega} = 1.2 \ \mu\text{A}$$

**1.27** The nominal values of $V_L$ and $I_L$ are given by

$$V_L = \frac{R_L}{R_S + R_L} V_S$$

$$I_L = \frac{V_S}{R_S + R_L}$$

After a 10% increase in $R_L$, the new values will be

$$V_L = \frac{1.1 R_L}{R_S + 1.1 R_L} V_S$$

$$I_L = \frac{V_S}{R_S + 1.1 R_L}$$

(a) The nominal values are

$$V_L = \frac{200}{5 + 200} \times 1 = 0.976 \text{ V}$$

$$I_L = \frac{1}{5 + 200} = 4.88 \mu\text{A}$$

After a 10% increase in $R_L$, the new values will be

$$V_L = \frac{1.1 \times 200}{5 + 1.1 \times 200} = 0.978 \text{ V}$$

$$I_L = \frac{1}{5 + 1.1 \times 200} = 4.44 \mu\text{A}$$

These values represent a 0.2% and 9% change, respectively. Since the load voltage remains relatively more constant than the load current, a Thévenin source is more appropriate here.

(b) The nominal values are

$$V_L = \frac{50}{5 + 50} \times 1 = 0.909 \text{ V}$$

$$I_L = \frac{1}{5 + 50} = 18.18 \text{ mA}$$

After a 10% increase in $R_L$, the new values will be

$$V_L = \frac{1.1 \times 50}{5 + 1.1 \times 50} = 0.917 \text{ V}$$

$$I_L = \frac{1}{5 + 1.1 \times 50} = 16.67 \text{ mA}$$

These values represent a 1% and 8% change, respectively. Since the load voltage remains relatively more constant than the load current, a Thévenin source is more appropriate here.

(c) The nominal values are

$$V_L = \frac{0.1}{2 + 0.1} \times 1 = 47.6 \text{ mV}$$

$$I_L = \frac{1}{2 + 0.1} = 0.476 \text{ mA}$$

After a 10% increase in $R_L$, the new values will be

$$V_L = \frac{1.1 \times 0.1}{2 + 1.1 \times 0.1} = 52.1 \text{ mV}$$

$$I_L = \frac{1}{2 + 1.1 \times 0.1} = 0.474 \text{ mA}$$

These values represent a 9% and 0.4% change, respectively. Since the load current remains relatively more constant than the load voltage, a Norton source is more appropriate here. The Norton equivalent current source is

$$I_S = \frac{V_S}{R_S} = \frac{1}{2} = 0.5 \text{ mA}$$

(d) The nominal values are

$$V_L = \frac{16}{150 + 16} \times 1 = 96.4 \text{ mV}$$

$$I_L = \frac{1}{150 + 16} = 6.02 \text{ mA}$$

After a 10% increase in $R_L$, the new values will be

$$V_L = \frac{1.1 \times 16}{150 + 1.1 \times 16} = 105 \text{ mV}$$

$$I_L = \frac{1}{150 + 1.1 \times 16} = 5.97 \text{ mA}$$

These values represent a 9% and 1% change, respectively. Since the load current remains relatively more constant than the load voltage, a Norton source is more appropriate here. The Norton equivalent current source is

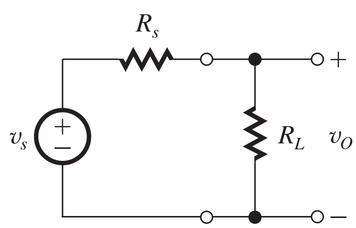
$$I_S = \frac{V_S}{R_S} = \frac{1}{150} = 6.67 \text{ mA}$$

**1.28**

$$P_L = v_O^2 \times \frac{1}{R_L}$$

$$= v_S^2 \frac{R_L^2}{(R_L + R_S)^2} \times \frac{1}{R_L}$$

$$= v_S^2 \frac{R_L}{(R_L + R_S)^2}$$




Since we are told that the power delivered to a 16Ω speaker load is 75% of the power delivered to a 32Ω speaker load,

$$P_L(R_L = 16\Omega) = 0.75 \times P_L(R_L = 32\Omega)$$

$$\frac{16}{(R_S + 32)^2} = 0.75 \times \frac{32}{(R_S + 32)^2}$$

$$\frac{\sqrt{16}}{R_S + 32} = \frac{\sqrt{24}}{R_S + 32}$$

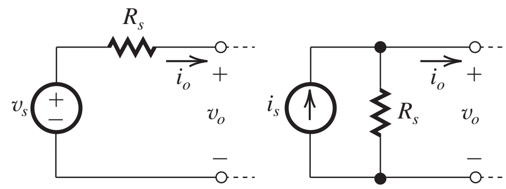
$$\Rightarrow (\sqrt{24} = \sqrt{16})R_S = \sqrt{16} \times 32 - \sqrt{24} \times 16$$

$$0.9R_S = 49.6$$

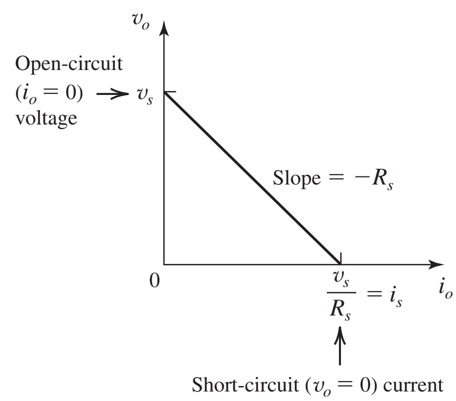
$$R_S = 55.2\Omega$$

**1.29** The observed output voltage is 1 mV/ °C, which is one half the voltage specified by the sensor, presumably under open-circuit conditions: that is, without a load connected. It follows that that sensor internal resistance must be equal to $R_L$, that is, 5 kΩ.

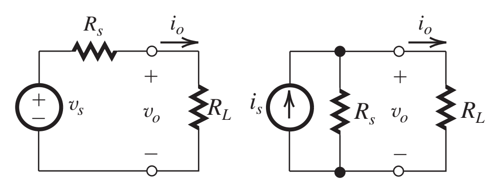
**1.30**




$v_o = v_s - i_o R_s$




**1.31**




$R_L$ represents the input resistance of the processor

For $v_o = 0.95v_s$

$$0.95 = \frac{R_L}{R_L + R_s} \Rightarrow R_L = 19R_s$$

For $i_o = 0.95i_s$

$$0.95 = \frac{R_s}{R_s + R_L} \Rightarrow R_L = R_S/19$$

**1.32**

| Case | $\omega$ (rad/s) | $f$(Hz) | $T$(s) |
|---|---|---|---|
| a | $3.14 \times 10^{10}$ | $5 \times 10^{9}$ | $0.2 \times 10^{-9}$ |
| b | $2 \times 10^{9}$ | $3.18 \times 10^{8}$ | $3.14 \times 10^{-9}$ |
| c | $6.28 \times 10^{10}$ | $1 \times 10^{10}$ | $1 \times 10^{-10}$ |
| d | $3.77 \times 10^{2}$ | 60 | $1.67 \times 10^{-2}$ |
| e | $6.28 \times 10^{4}$ | $1 \times 10^{4}$ | $1 \times 10^{-4}$ |
| f | $6.28 \times 10^{5}$ | $1 \times 10^{5}$ | $1 \times 10^{-5}$ |

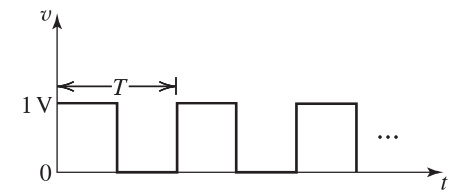
**1.33** (a) $v = 10\sin(2\pi \times 10^3 t)$, V

(b) $v = 120\sqrt{2}\sin(2\pi \times 60)$, V

(c) $v = 0.1\sin(2000t)$, V

(d) $v = 0.1\sin(2\pi \times 10^3 t)$, V

**1.34** Comparing the given waveform to that described by Eq. (1.2), we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform looks as follows:




Average value = 0.5 V
Peak-to-peak value = 1 V
Lowest value = 0 V
Highest value = 1 V

Period $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3}$ s

Frequency $f = \frac{1}{I} = 1$ kHz

**1.35** (a) $V_{peak} = 117 \times \sqrt{2} = 165$ V

(b) $V_{rms} = 33.9/\sqrt{2} = 24$ V

(c) $V_{peak} = 220 \times \sqrt{2} = 311$ V

(d) $V_{peak} = 220 \times \sqrt{2} = 311$ kV

**1.36** The two harmonics have the ratio 126/98 = 9/7. Thus, these are the 7th and 9th harmonics. From Eq. (1.2), we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of 98/7, or 14 kHz, and peak amplitude of $63 \times 7 = 441$ mV. The rms value of the fundamental will be $441/\sqrt{2} = 312$ mV. To find the peak-to-peak amplitude of the square wave, we note that $4V/\pi = 441$ mV. Thus,

Peak-to-peak amplitude

$= 2V = 441 \times \frac{\pi}{2} = 693$ mV

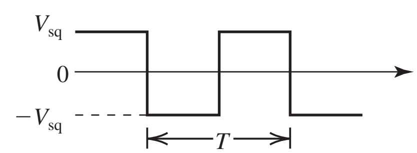
Period $T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4\ \mu$s

**1.37** The rms value of a symmetrical square wave with peak amplitude $\hat{V}$ is simply $\hat{V}$. Taking the root-mean-square of the first 5 sinusoidal terms in Eq. (1.2) gives an rms value of,

$$\frac{4\hat{V}}{\pi\sqrt{2}}\sqrt{1^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{9}\right)^2}$$
$$= 0.980\hat{V}$$

which is 2% lower than the rms value of the square wave.

**1.38** If the amplitude of the square wave is $V_{sq}$, then the power delivered by the square wave to a resistance $R$ will be $V_{sq}^2/R$. If this power is to be equal to that delivered by a sine wave of peak amplitude $\hat{V}$, then




$$\frac{V_{sq}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus, $V_{sq} = \hat{V}/\sqrt{2}$. This result is independent of frequency.

**1.39**

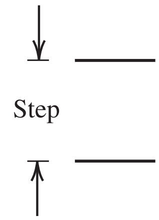
| Decimal | Binary |
|---|---|
| 0 | 0 |
| 6 | 110 |
| 11 | 1011 |
| 28 | 11100 |
| 59 | 111011 |

**1.40** (a) For $N$ bits there will be $2^N$ possible levels, from 0 to $V_{FS}$. Thus there will be $(2^N - 1)$ discrete steps from 0 to $V_{FS}$ with the step size given by

Step size $= \frac{V_{FS}}{2^N - 1}$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is




$$\frac{1}{2} \times \text{ step size} = \frac{1}{2}\frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

(c) $\dfrac{5\text{ V}}{2^N - 1} \leq 2\text{ mV}$

$2^N - 1 \geq 2500$

$2^N \geq 2501 \Rightarrow N = 12,$

For $N = 12$,

$$\text{Resolution} = \frac{5}{2^{12} - 1} = 1.2\text{ mV}$$

$$\text{Quantization error} = \frac{1.2}{2} = 0.6\text{ mV}$$

**1.41**

| $b_3$ | $b_2$ | $b_1$ | $b_0$ | Value Represented |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | +0 |
| 0 | 0 | 0 | 1 | +1 |
| 0 | 0 | 1 | 0 | +2 |
| 0 | 0 | 1 | 1 | +3 |
| 0 | 1 | 0 | 0 | +4 |
| 0 | 1 | 0 | 1 | +5 |
| 0 | 1 | 1 | 0 | +6 |
| 0 | 1 | 1 | 1 | +7 |
| 1 | 0 | 0 | 0 | –0 |
| 1 | 0 | 0 | 1 | –1 |
| 1 | 0 | 1 | 0 | –2 |
| 1 | 0 | 1 | 1 | –3 |
| 1 | 1 | 0 | 0 | –4 |
| 1 | 1 | 0 | 1 | –5 |
| 1 | 1 | 1 | 0 | –6 |
| 1 | 1 | 1 | 1 | –7 |

Note that there are two possible representations of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range ±3.5 V can be represented.

| Input | Steps | Code |
|---|---|---|
| +2.5 V | +5 | 0101 |
| −3.0 V | −6 | 1110 |
| +2.7 | +5 | 0101 |
| −2.8 | −6 | 1110 |

**1.42** (a) When $b_i = 1$, the $i$th switch is in position 1 and a current $(V_{ref}/2^i R)$ flows to the output. Thus $i_O$ will be the sum of all the currents corresponding to "1" bits, that is,

$$i_O = \frac{V_{\text{ref}}}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N}\right)$$

(b) $b_N$ is the LSB

$b_1$ is the MSB

(c) $$i_{O\max} = \frac{10\text{ V}}{10\text{ k}\Omega}\left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8}\right)$$

$= 0.99609375$ mA

Corresponding to the LSB changing from 0 to 1 the output changes by $(10/10) \times 1/2^8 =$ 3.91 µA.

**1.43** There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be $44,100 \times 16 = 7.056 \times 10^5$ bits per second.

**1.44** Each pixel requires $8 + 8 + 8 = 24$ bits to represent it. We will approximate a megapixel as $10^6$ pixels, and a Gbit as $10^9$ bits. Thus, each image requires $24 \times 10 \times 10^6 = 2.4 \times 10^8$ bits. The number of such images that fit in 16 Gbits of memory is

$$\left\lfloor\frac{2.4 \times 10^8}{16 \times 10^9}\right\rfloor = \lfloor 66.7 \rfloor = 66$$

**1.45** (a) $A_v = \dfrac{v_O}{v_I} = \dfrac{10\text{ V}}{100\text{ mV}} = 100\text{ V/V}$

or $20 \log 100 = 40$ dB

$$A_i = \frac{i_O}{i_I} = \frac{v_O/R_L}{i_I} = \frac{10\text{ V}/100\ \Omega}{100\ \mu\text{A}} = \frac{0.1\text{ A}}{100\ \mu\text{A}}$$

$= 1000$ A/A

or $20 \log 1000 = 60$ dB

$$A_p = \frac{v_O i_O}{v_I i_I} = \frac{v_O}{v_I} \times \frac{i_O}{i_I} = 100 \times 1000$$

$= 10^5$ W/W

or $10 \log 10^5 = 50$ dB

(b) $A_v = \dfrac{v_O}{v_I} = \dfrac{1 \text{ V}}{10 \ \mu\text{V}} = 1 \times 10^5$ V/V

or $20 \log 1 \times 10^5 = 100$ dB

$$A_i = \frac{i_O}{i_I} = \frac{v_O/R_L}{i_I} = \frac{1 \text{ V}/10 \text{ k}\Omega}{100 \text{ nA}}$$

$$= \frac{0.1 \text{ mA}}{100 \text{ nA}} = \frac{0.1 \times 10^{-3}}{100 \times 10^{-9}} = 1000 \text{ A/A}$$

or $20 \log A_i = 60$ dB

$$A_p = \frac{v_O i_O}{v_I i_I} = \frac{v_O}{v_I} \times \frac{i_O}{i_I}$$

$= 1 \times 10^5 \times 1000$

$= 1 \times 10^8$ W/W

or $10 \log A_P = 80$ dB

(c) $A_v = \dfrac{v_O}{v_i} = \dfrac{5 \text{ V}}{1 \text{ V}} = 5$ V/V

or $20 \log 5 = 14$ dB

$$A_i = \frac{i_O}{i_I} = \frac{v_O/R_L}{i_I} = \frac{5 \text{ V}/10 \ \Omega}{1 \text{ mA}}$$

$$= \frac{0.5 \text{ A}}{1 \text{ mA}} = 500 \text{ A/A}$$

or $20 \log 500 = 54$ dB

$$A_p = \frac{v_O i_O}{v_I i_I} = \frac{v_O}{v_I} \times \frac{i_O}{i_I}$$

$= 5 \times 500 = 2500$ W/W

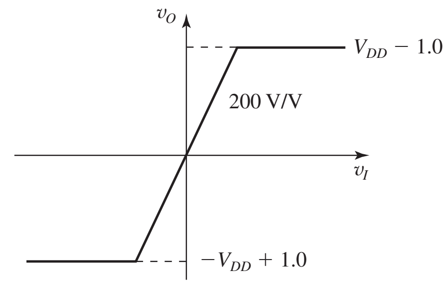
or $10 \log A_p = 34$ dB

**1.46** For $\pm 5$ V supplies:

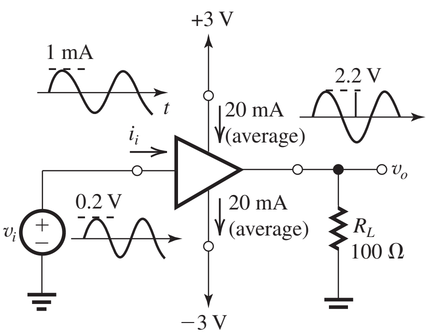
The largest undistorted sine-wave output is of 4-V peak amplitude or $4/\sqrt{2} = 2.8$ $V_{rms}$. Input needed is 14 $mV_{rms}$.

For $\pm 10$-V supplies, the largest undistorted sine-wave output is of 9-V peak amplitude or 6.4 $V_{rms}$. Input needed is 32 $mV_{rms}$.

For $\pm 15$-V supplies, the largest undistorted sine-wave output is of 14-V peak amplitude or 9.9 $V_{rms}$. The input needed is 9.9 V/200 = 49.5 $mV_{rms}$.




**1.47**




$$A_v = \frac{v_o}{v_i} = \frac{2.2}{0.2}$$

$= 11$ V/V

or $20 \log 11 = 20.8$ dB

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V}/100 \ \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

or $20 \log A_i = 26.8$ dB

$$A_p = \frac{p_o}{p_i} = \frac{(2.2/\sqrt{2})^2/100}{\dfrac{0.2}{\sqrt{2}} \times \dfrac{10^{-3}}{\sqrt{2}}}$$

$= 242$ W/W

or $10 \log A_P = 23.8$ dB

Supply power $= 2 \times 3 \text{ V} \times 20 \text{ mA} = 120$ mW

Output power $= \dfrac{v_{orms}^2}{R_L} = \dfrac{(2.2/\sqrt{2})^2}{100 \ \Omega} = 24.2$ mW

Input power $= \dfrac{24.2}{242} = 0.1$ mW (negligible)

Amplifier dissipation $\simeq$ Supply power − Output power

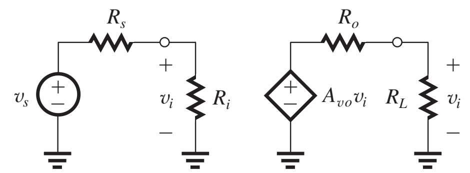
$= 120 - 24.2 = 95.8$ mW

$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

**1.48** $v_o = A_{vo} v_i \dfrac{R_L}{R_L + R_o}$

$$= A_{vo}\left(v_s \frac{R_i}{R_i + R_s}\right)\frac{R_L}{R_L + R_o}$$




Thus,

$$\frac{v_o}{v_s} = A_{vo}\frac{R_i}{R_i + R_s}\frac{R_L}{R_L + R_o}$$

(a) $A_{vo} = 100, R_i = 10R_s, R_L = 10R_o$:

$$\frac{v_o}{v_s} = 100 \times \frac{10R_s}{10R_s + R_s} \times \frac{10R_o}{10R_o + R_o}$$

$= 82.6$ V/V or $20 \log 82.6 = 38.3$ dB

(b) $A_{vo} = 100, R_i = R_s, R_L = R_o$:

$$\frac{v_o}{v_s} = 100 \times \frac{1}{2} \times \frac{1}{2} = 25 \text{ V/V or } 20 \log 25 = 28 \text{ dB}$$

(c) $A_{vo} = 100$ V/V, $R_i = R_s/10, R_L = R_o/10$:

$$\frac{v_o}{v_s} = 100\frac{R_s/10}{(R_s/10) + R_s}\ \frac{R_o/10}{(R_o/10) + R_o}$$

$= 0.826$ V/V or $20 \log 0.826 = -1.7$ dB

**1.49** (a)

$$\frac{v_o}{v_s} = \frac{v_i}{v_s} \times \frac{v_o}{v_i}$$

$$= \frac{1}{5+1} \times 100 \times \frac{100}{200+100}$$

$= 5.56$ V/V

Much of the amplifier's 100 V/V gain is lost in the source resistance and amplifier's output resistance. If the source were connected directly to the load, the gain would be

$$\frac{v_o}{v_s} = \frac{0.1}{5 + 0.1} = 0.0196 \text{ V/V}$$

This is a factor of 284× smaller than the gain with the amplifier in place!

(b)

The equivalent current amplifier has a dependent current source with a value of

$$\frac{100 \text{ V/V}}{200\Omega} \times i_i = \frac{100 \text{ V/V}}{200\Omega} \times 1000\Omega \times v_i$$

$= 500 \times i_i$

Thus, $\dfrac{i_o}{i_s} = \dfrac{i_i}{i_s} \times \dfrac{i_o}{i_i}$

$$= \frac{5}{5+1} \times 500 \times \frac{200}{200+100}$$

$= 277.8$ A/A

Using the voltage amplifier model, the current gain can be found as follows,

$$\frac{i_o}{i_s} = \frac{i_i}{i_s} \times \frac{v_i}{i_i} \times \frac{i_o}{v_i}$$

$$= \frac{5}{5+1} \times 1000 \times \frac{100 \text{ V/V}}{200+100}$$

$= 277.8$ A/A

**1.50** In Example 1.3, when the first and the second stages are interchanged, the circuit looks like the figure above, and

$$\frac{v_{i1}}{v_s} = \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 100 \text{ k}\Omega} = 0.5 \text{ V/V}$$

This figure belongs to Problem **1.49**.

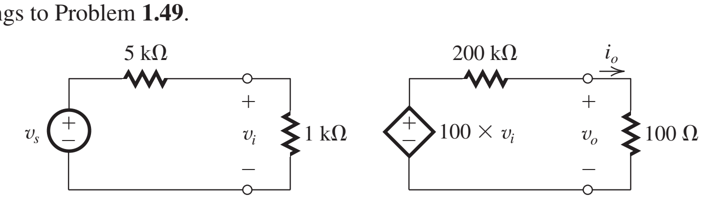



Figure 1

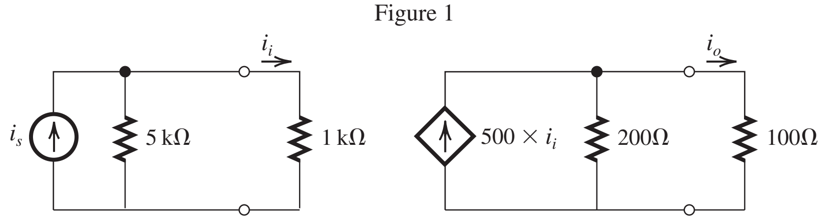
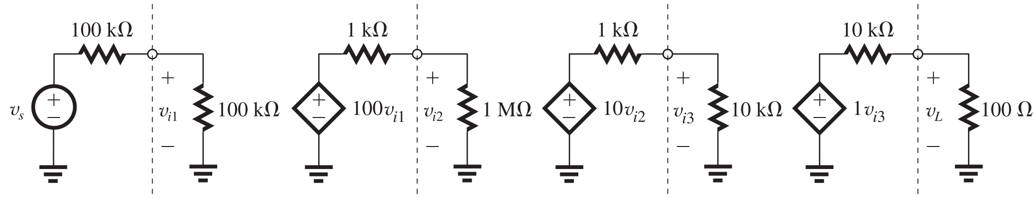



Figure 2

This figure belongs to Problem **1.50**.




$$A_{v1} = \frac{v_{i2}}{v_{i1}} = 100 \times \frac{1\text{ M}\Omega}{1\text{ M}\Omega + 1\text{ k}\Omega}$$

$= 99.9$ V/V

$$A_{v2} = \frac{v_{i3}}{v_{i2}} = 10 \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 1\text{ k}\Omega}$$

$= 9.09$ V/V

$$A_{v3} = \frac{v_L}{v_{i3}} = 1 \times \frac{100\ \Omega}{100\ \Omega + 10\ \Omega} = 0.909\text{ V/V}$$

$$\text{Total gain} = A_v = \frac{v_L}{v_{i1}} = A_{v1} \times A_{v2} \times A_{v3}$$

$= 99.9 \times 9.09 \times 0.909 = 825.5$ V/V

The voltage gain from source to load is

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i1}} \times \frac{v_{i1}}{v_S} = A_v \cdot \frac{v_{i1}}{v_S}$$

$= 825.5 \times 0.5$

$= 412.7$ V/V

The overall voltage has reduced appreciably. This is because the input resistance of the first stage, $R_{in}$, is comparable to the source resistance $R_s$. In Example 1.3 the input resistance of the first stage is much larger than the source resistance.

**1.51** The equivalent circuit at the output side of a current amplifier loaded with a resistance $R_L$ is shown. Since

$$i_o = (A_{is}i_i)\frac{R_o}{R_o + R_L}$$

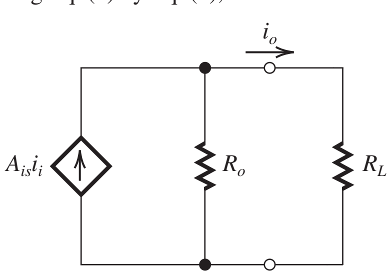
we can write

$$1 = (A_{is}i_i)\frac{R_o}{R_o + 1} \qquad (1)$$

and

$$0.5 = (A_{is}i_i)\frac{R_o}{R_o + 12} \qquad (2)$$

Dividing Eq. (1) by Eq. (2), we have




$$2 = \frac{R_o + 12}{R_o + 1} \Rightarrow R_o = 10\text{ k}\Omega$$

$$A_{is}i_i = 1 \times \frac{10 + 1}{10} = 1.1\text{ mA}$$

**1.52**

The current gain is

$$\frac{i_o}{i_i} = \frac{R_m}{R_o + R_L}$$

$$= \frac{5000}{10 + 1000}$$

$= 4.95$ A/A $= 13.9$ dB

The voltage gain is

$$\frac{v_o}{v_s} = \frac{i_i}{v_s} \times \frac{i_o}{i_i} \times \frac{v_o}{i_o}$$

$$= \frac{1}{R_s + R_i} \times \frac{i_o}{i_i} \times R_L$$

$$= \frac{1}{1000 + 100} \times 4.95 \times 1000$$

$= 4.90$ V/V $= 13.8$ dB

The power gain is

$$\frac{v_o i_o}{v_s i_i} = 4.95 \times 4.90$$

$= 24.3$ W/W $= 27.7$ dB

**1.53**

$G_m = 60$ mA/V

$R_o = 20$ kΩ

$R_L = 1$ kΩ

$$v_i = v_s \frac{R_i}{R_s + R_i}$$

$$= v_s \frac{2}{1 + 2} = \frac{2}{3} v_s$$

$$v_o = G_m v_i (R_L \parallel R_o)$$

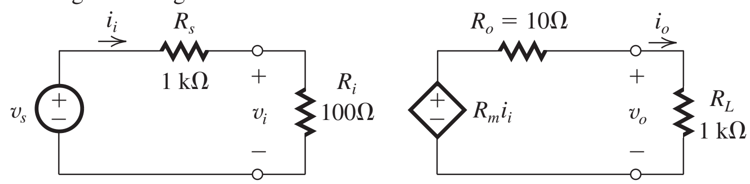
$$= 60 \frac{20 \times 1}{20 + 1} v_i$$

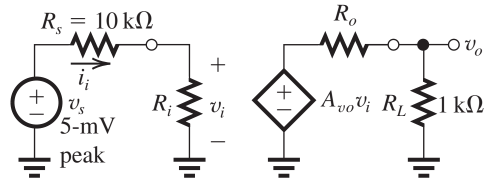
$$= 60 \frac{20}{21} \times \frac{2}{3} v_s$$

$$\text{Overall voltage gain} \equiv \frac{v_o}{v_s} = 38.1\text{ V/V}$$

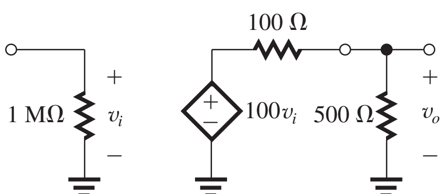
 

This figure belongs to Problem **1.52**.




**1.54**




$20 \log A_{vo} = 40\text{ dB} \Rightarrow A_{vo} = 100\text{ V/V}$

$$A_v = \frac{v_o}{v_i}$$

$$= 100 \times \frac{500}{500 + 100}$$

$= 83.3$ V/V

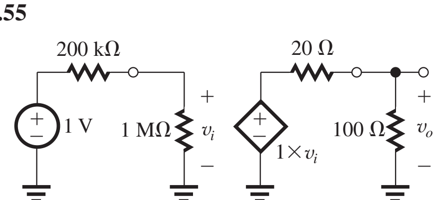
or $20 \log 83.3 = 38.4$ dB

$$A_p = \frac{v_o^2/500\ \Omega}{v_i^2/1\ \text{M}\Omega} = A_v^2 \times 10^4 = 1.39 \times 10^7\text{ W/W}$$

or $10 \log (1.39 \times 10^7) = 71.4$ dB.

For a peak output sine-wave current of 20 mA, the peak output voltage will be 20 mA × 500 Ω = 10 V. Correspondingly $v_i$ will be a sine wave with a peak value of $10\text{ V}/A_v = 10/83.3$, or an rms value of $10/(83.3 \times \sqrt{2}) = 0.085$ V. Corresponding output power $= (10/\sqrt{2})^2/500\ \Omega = 0.1$ W

**1.55**




$$v_o = 1\text{ V} \times \frac{1\text{ M}\Omega}{1\text{ M}\Omega + 200\text{ k}\Omega}$$

$$\times 1 \times \frac{100\ \Omega}{100\ \Omega + 20\ \Omega}$$

$$= \frac{1}{1.2} \times \frac{100}{120} = 0.69\text{ V}$$

$$\text{Voltage gain} = \frac{v_o}{v_s} = 0.69\text{ V/V or } -3.2\text{ dB}$$

$$\text{Current gain} = \frac{v_o/100\ \Omega}{v_s/1.2\text{ M}\Omega} = 0.69 \times 1.2 \times 10^4$$

$= 8280$ A/A  or  78.4 dB

$$\text{Power gain} = \frac{v_o^2/100\ \Omega}{v_s^2/1.2\text{ M}\Omega} = 5713\text{ W/W}$$

or $10 \log 5713 = 37.6$ dB

(This takes into account the power dissipated in the internal resistance of the source.)

**1.56** (a) Case S-A-B-L (see figure on next page):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ib}} \times \frac{v_{ib}}{v_{ia}} \times \frac{v_{ia}}{v_s} =$$

$$\left(10 \times \frac{100}{100 + 1000}\right) \times \left(100 \times \frac{10}{10 + 10}\right) \times \left(\frac{100}{100 + 100}\right)$$

$\dfrac{v_o}{v_s} = 22.7$ V/V and gain in dB $20 \log 22.7 =$

27.1 dB

(b) Case S-B-A-L (see figure on next page):

$$\frac{v_o}{v_s} = \frac{v_o}{v_{ia}} \cdot \frac{v_{ia}}{v_{ib}} \cdot \frac{v_{ib}}{v_s}$$

$$= \left(100 \times \frac{100}{100 + 10\text{ K}}\right) \times \left(10 \times \frac{100\text{ K}}{100\text{ K} + 1\text{ K}}\right) \times \left(\frac{10\text{ K}}{10\text{ K} + 100\text{ K}}\right)$$

$\dfrac{v_o}{v_s} = 0.89$ V/V and gain in dB is $20 \log 0.89 = -1$ dB. Obviously, case a is preferred because it provides higher voltage gain.

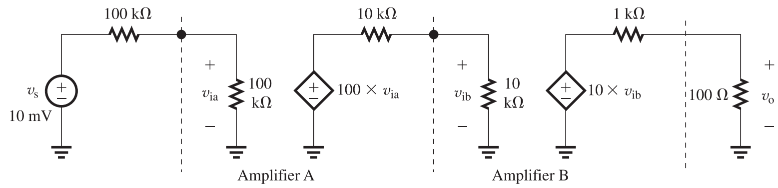
**1.57** Each of stages #1, 2, ..., $(n - 1)$ can be represented by the equivalent circuit:

$$\frac{v_o}{v_s} = \frac{v_{i1}}{v_s} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i3}}{v_{i2}} \times \cdots \times \frac{v_{in}}{v_{i(n-1)}} \times \frac{v_o}{v_{in}}$$

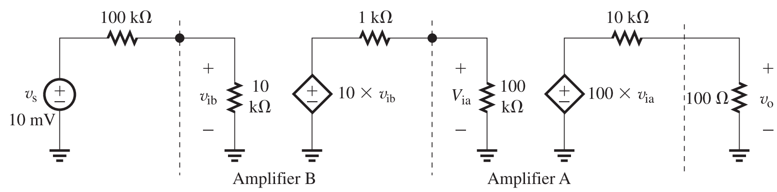
where

This figure belongs to **1.56**, part (a).




This figure belongs to **1.56**, part (b).




$$\frac{v_{i1}}{v_s} = \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega} = 0.5\text{ V/V}$$

$$\frac{v_o}{v_{in}} = 10 \times \frac{200\ \Omega}{1\text{ k}\Omega + 200\ \Omega} = 1.67\text{ V/V}$$

$$\frac{v_{i2}}{v_{i1}} = \frac{v_{i3}}{v_{i2}} = \cdots = \frac{v_{in}}{v_{i(n-1)}} = 10 \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega}$$

$$= 9.09\text{ V/V}$$

Thus,

$$\frac{v_o}{v_s} = 0.5 \times (9.09)^{n-1} \times 1.67 = 0.833 \times (9.09)^{n-1}$$

For $v_s = 5$ mV and $v_o = 3$ V, the gain $\frac{v_o}{v_s}$ must be $\geq 600$, thus

$$0.833 \times (9.09)^{n-1} \geq 600$$

$$\Rightarrow\ n = 4$$

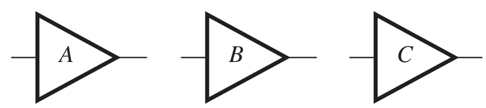
Thus four amplifier stages are needed, resulting in

$$\frac{v_o}{v_s} = 0.833 \times (9.09)^3 = 625.7\text{ V/V}$$

and correspondingly

$$v_o = 625.7 \times 5\text{ mV} = 3.13\text{ V}$$

**1.58** Deliver 0.5 W to a 100-Ω load.
Source is 30 mV rms with 0.5-MΩ source resistance. Choose from these three amplifier types:



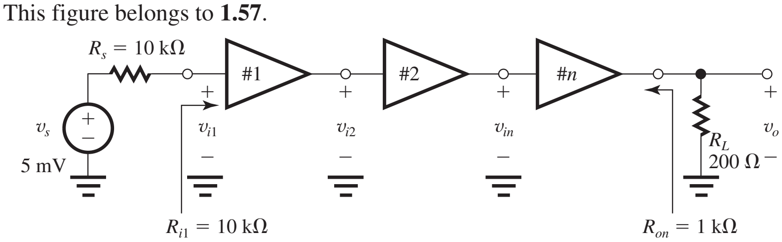

| A | B | C |
|---|---|---|
| $R_i = 1\text{ M}\Omega$ | $R_i = 10\text{ k}\Omega$ | $R_i = 10\text{ k}\Omega$ |
| $A_v = 10\text{ V/V}$ | $A_v = 100\text{ V/V}$ | $A_v = 1\text{ V/V}$ |
| $R_o = 10\text{ k}\Omega$ | $R_o = 1\text{ k}\Omega$ | $R_o = 20\ \Omega$ |

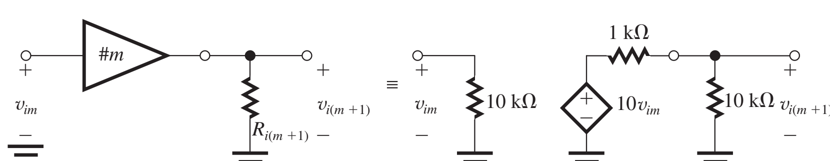
Choose order to eliminate loading on input and output:
*A*, first, to minimize loading on 0.5-MΩ source
*B*, second, to boost gain
*C*, third, to minimize loading at 100-Ω output.

We first attempt a cascade of the three stages in the order *A*, *B*, *C* (see figure above), and obtain

$$\frac{v_{i1}}{v_s} = \frac{1\text{ M}\Omega}{1\text{ M}\Omega + 0.5\text{ M}\Omega} = \frac{1}{1.5}$$

This figure belongs to **1.57**.



$$\Rightarrow v_{i1} = 30 \times \frac{1}{1.5} = 20 \text{ mV}$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 10 \text{ k}\Omega} = 5$$

$$\Rightarrow v_{i2} = 20 \times 5 = 100 \text{ mV}$$

$$\frac{v_{i3}}{v_{i2}} = 100 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 1 \text{ k}\Omega} = 90.9$$

$$\Rightarrow v_{i3} = 100 \text{ mV} \times 90.9 = 9.09 \text{ V}$$

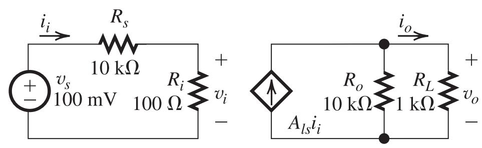
$$\frac{v_o}{v_{i3}} = 1 \times \frac{100\ \Omega}{100\ \Omega + 20\ \Omega} = 0.833$$

$$\Rightarrow v_o = 9.09 \times 0.833 = 7.6 \text{ V}$$

$$P_o = \frac{v_{orms}^2}{R_L} = \frac{7.6^2}{100} = 0.57 \text{ W}$$

which exceeds the required 0.5 W. Also, the signal throughout the amplifier chain never drops below 20 mV (which is greater than the required minimum of 10 mV).

**1.59**




(a) Current gain $= \dfrac{i_o}{i_i}$

$$= A_{is}\frac{R_o}{R_o + R_L}$$

$$= 100\frac{10}{11}$$

$$= 90.9 \text{ A/A} = 39.2 \text{ dB}$$

(b) Voltage gain $= \dfrac{v_o}{v_s} = \dfrac{i_o R_L}{i_i(R_s + R_i)}$

$$= \frac{i_o}{i_i}\frac{R_L}{R_s + R_i}$$

$$= 90.9 \times \frac{1}{10 + 0.1}$$

$$= 9 \text{ V/V} = 19.1 \text{ dB}$$

(c) Power gain $= A_p = \dfrac{v_o i_o}{v_s i_i}$

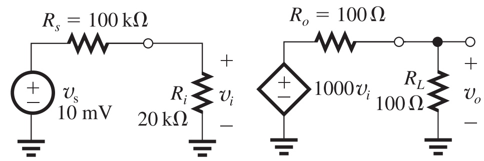
$$= 9 \times 90.9$$

$$= 818 \text{ W/W} = 29.1 \text{ dB}$$

**1.60**

(a)

$$v_o = 10 \text{ mV} \times \frac{20}{20 + 100} \times 1000 \times \frac{100}{100 + 100}$$

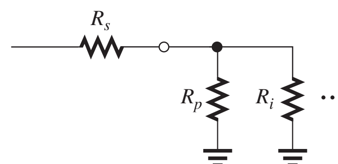



$$= 833 \text{ mV}$$

(b) $\dfrac{v_o}{v_s} = \dfrac{833 \text{ mV}}{10 \text{ mV}} = 83.3 \text{ V/V}$

(c) $\dfrac{v_o}{v_i} = 1000 \times \dfrac{100}{100 + 100} = 500 \text{ V/V}$

(d)




Connect a resistance $R_P$ in parallel with the input and select its value from

$$\frac{(R_p \parallel R_i)}{(R_p \parallel R_i) + R_s} = \frac{1}{2}\frac{R_i}{R_i + R_s}$$

$$\Rightarrow 1 + \frac{R_s}{R_p \parallel R_i} = 12 \Rightarrow R_p \parallel R_i = \frac{R_S}{11} = \frac{100}{11}$$

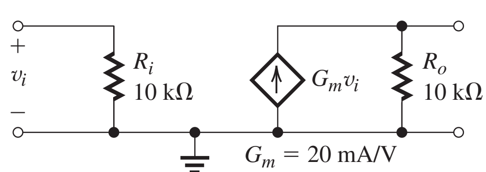
$$\Rightarrow \frac{1}{R_p} + \frac{1}{R_i} = \frac{11}{100}$$

$$R_p = \frac{1}{0.11 - 0.05} = 16.7 \text{ k}\Omega$$

**1.61** To obtain the weighted sum of $v_1$ and $v_2$

$$v_o = 10v_1 + 20v_2$$

we use two transconductance amplifiers and sum their output currents. Each transconductance amplifier has the following equivalent circuit:



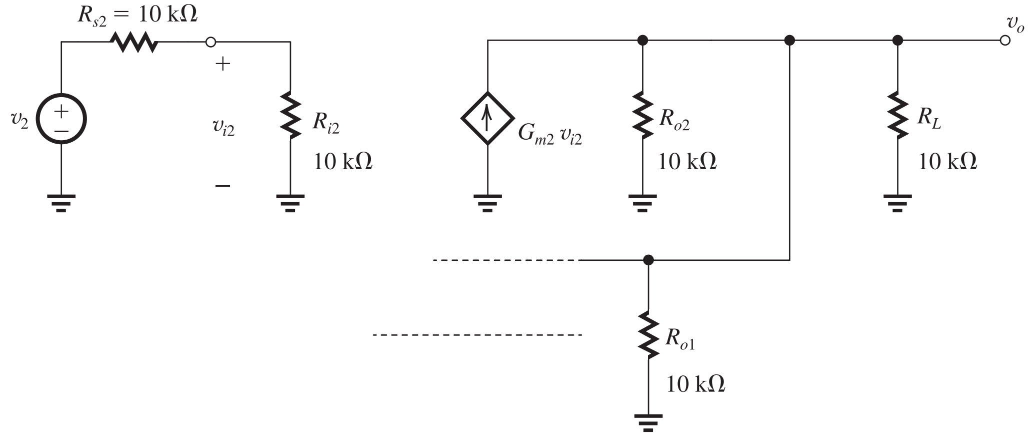

Consider first the path for the signal requiring higher gain, namely $v_2$. See figure at top of next page.

The parallel connection of the two amplifiers at the output and the connection of $R_L$ means that the total resistance at the output is

$$10 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = \frac{10}{3} \text{ k}\Omega.$$

This figure belongs to Problem **1.61**.




Thus the component of $v_o$ due to $v_2$ will be

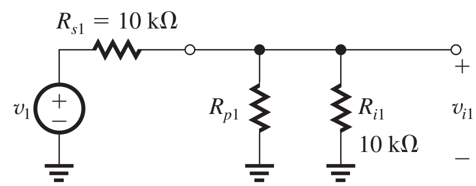
$$v_{o2} = v_2 \frac{10}{10+10} \times G_{m2} \times \frac{10}{3}$$

$$= v_2 \times 0.5 \times 20 \times \frac{10}{3} = 33.3 v_2$$

To reduce the gain seen by $v_2$ from 33.3 to 20, we connect a resistance $R_p$ in parallel with $R_L$,

$$\left(\frac{10}{3} \parallel R_p\right) = 2\ k\Omega \Rightarrow R_p = 5\ k\Omega$$

We next consider the path for $v_1$. Since $v_1$ must see a gain factor of only 10, which is half that seen by $v_2$, we have to reduce the fraction of $v_1$ that appears at the input of its transconductance amplifier to half that that appears at the input of the $v_2$ transconductance amplifier. We just saw that 0.5 $v_2$ appears at the input of the $v_2$ transconductance amplifier. Thus, for the $v_1$ transconductance amplifier, we want $0.25 v_1$ to appear at the input. This can be achieved by shunting the input of the $v_1$ transconductance amplifier by a resistance $R_{p1}$ as in the figure in the next column.




The value of $R_{p1}$ can be found from

$$\frac{(R_{p1} \parallel R_{i1})}{(R_{p1} \parallel R_{i1}) + R_{s1}} = 0.25$$

Thus,

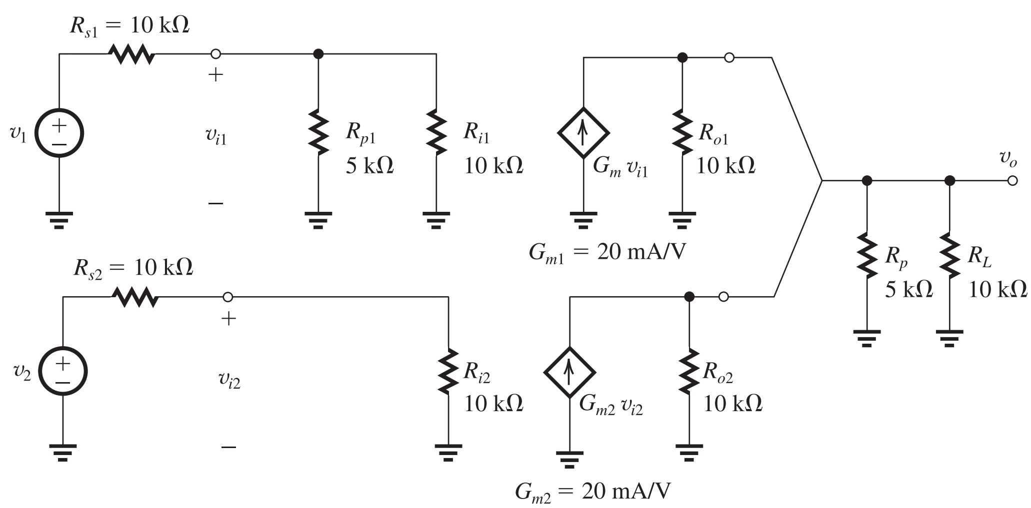
$$1 + \frac{R_{s1}}{(R_{p1} \parallel R_{i1})} = 4$$

$$\Rightarrow R_{p1} \parallel R_{i1} = \frac{R_{s1}}{3} = \frac{10}{3}$$

$$R_{p1} \parallel 10 = \frac{10}{3}$$

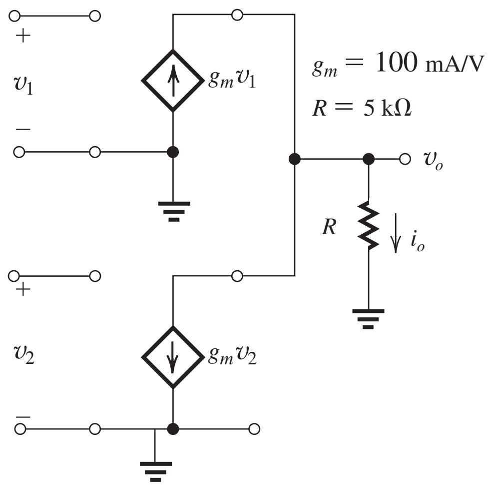
$$\Rightarrow R_{p1} = 5\ k\Omega$$

The final circuit will be as follows:



**1.62**



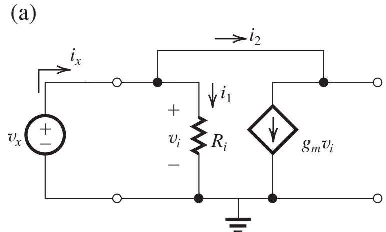

$i_o = g_m v_1 - g_m v_2$

$v_o = i_o R_L = g_m R(v_1 - v_2)$

$v_1 = v_2 = 1$ V $\quad \therefore v_o = 0$ V

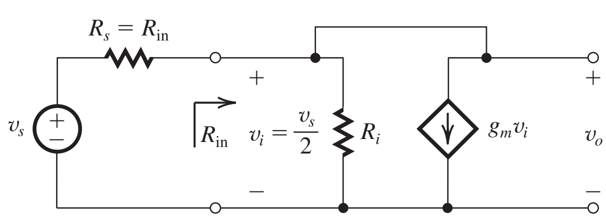
$$\left.\begin{matrix} v_1 = 1.01\text{ V} \\ v_2 = 0.99\text{ V}\end{matrix}\right\} \quad \therefore v_o = 100 \times 5 \times 0.02 = 10\text{ V}$$

**1.63** (a)




$$\left.\begin{aligned} i_x &= i_1 + i_2 \\ i_1 &= v_i/R_i \\ i_2 &= g_m v_i \\ v_i &= v_x \end{aligned}\right\} \quad \begin{aligned} i_x &= v_x/R_i + g_m v_x \\ i_x &= v_x\left(\frac{1}{R_i} + g_m\right) \\ \frac{v_x}{i_x} &= \frac{1}{1/R_i + g_m} \\ &= \frac{R_i}{1 + g_m R_i} = R_{\text{in}} \end{aligned}$$

(b)



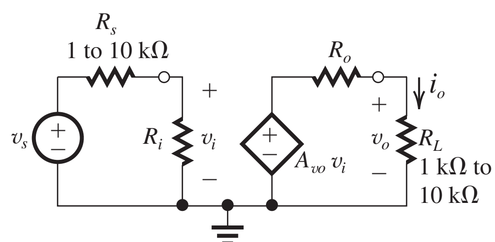

When driven by a source with source resistance $R_{in}$ as shown in the figure above,

$$v_i = \frac{R_{in}}{R_s + R_{in}} \times v_s = \frac{R_{in}}{R_{in} + R_{in}} \times v_s = 0.5 \times v_s$$

Thus,

$$\frac{v_o}{v_s} = 0.5\frac{v_o}{v_i}$$

**1.64** Voltage amplifier:




For $R_s$ varying in the range 1 kΩ to 10 kΩ and $\Delta v_o$ limited to 10%, select $R_i$ to be sufficiently large:

$R_i \geq 10\, R_{s\max}$

$R_i = 10 \times 10\text{ k}\Omega = 100\text{ k}\Omega = 1 \times 10^5\ \Omega$

For $R_L$ varying in the range 1 kΩ to 10 kΩ, the load voltage variation limited to 10%, select $R_o$ sufficiently low:

$$R_o \leq \frac{R_{L\min}}{10}$$

$$R_o = \frac{1\text{ k}\Omega}{10} = 100\ \Omega = 1 \times 10^2\ \Omega$$

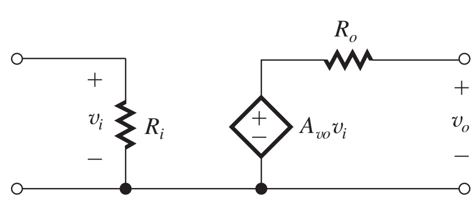
Now find $A_{vo}$:

$$v_{o\min} = 10\text{ mV} \times \frac{R_i}{R_i + R_{s\max}} \times A_{vo}\frac{R_{L\min}}{R_o + R_{L\min}}$$

$$1 = 10 \times 10^{-3} \times \frac{100\text{ k}\Omega}{100\text{ k}\Omega + 10\text{ k}\Omega}$$

$$\times A_{vo} \times \frac{1\text{ k}\Omega}{100\ \Omega + 1\text{ k}\Omega}$$

$\Rightarrow A_{vo} = 121$ V/V



Values for the voltage amplifier equivalent circuit are

$R_i = 1 \times 10^5\ \Omega,\ A_{vo} = 121\ \text{V/V},$ and

$R_o = 1 \times 10^2\ \Omega$

**1.65** Transresistance amplifier:

To limit $\Delta v_o$ to 10% corresponding to $R_s$ varying in the range 1 kΩ to 10 kΩ, we select $R_i$ sufficiently low;

$$R_i \le \frac{R_{s\min}}{10}$$

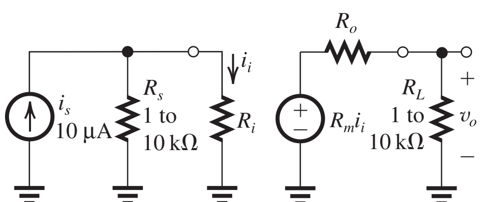
Thus, $R_i = 100\ \Omega = 1 \times 10^2\ \Omega$

To limit $\Delta v_o$ to 10% while $R_L$ varies over the range 1 kΩ to 10 kΩ, we select $R_o$ sufficiently low;

$$R_o \le \frac{R_{L\min}}{10}$$

Thus, $R_o = 100\ \Omega = 1 \times 10^2\ \Omega$

Now, for $i_s = 10\ \mu\text{A}$,

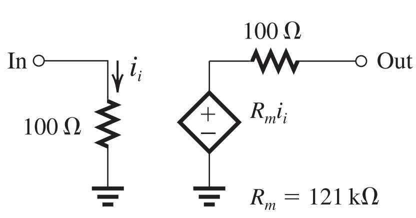



$$v_{o\min} = 10^{-5}\frac{R_{s\min}}{R_{s\min}+R_i}R_m\frac{R_{L\min}}{R_{L\min}+R_o}$$

$$1 = 10^{-5}\frac{1000}{1000+100}R_m\frac{1000}{1000+100}$$

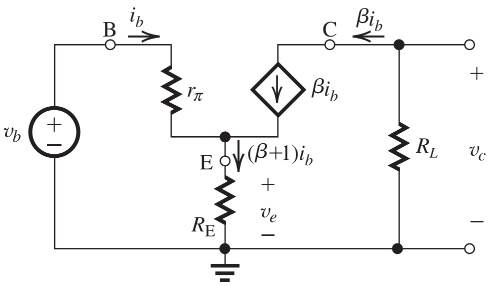
$\Rightarrow R_m = 1.21 \times 10^5\ \Omega$

$= 121\ \text{k}\Omega$




**1.66**

The node equation at $E$ yields the current through $R_E$ as $(\beta i_b + i_b) = (\beta+1)i_b$. The voltage $v_c$ can be found in terms of $i_b$ as




$$v_c = -\beta i_b R_L \qquad (1)$$

The voltage $v_b$ can be related to $i_b$ by writing for the input loop:

$$v_b = i_b r_\pi + (\beta+1)i_b R_E$$

Thus,

$$v_b = [r_\pi + (\beta+1)R_E]i_b \qquad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$\frac{v_c}{v_b} = -\frac{\beta R_L}{r_\pi + (\beta+1)R_E} \qquad \text{Q.E.D}$$

The voltage $v_e$ is related to $i_b$ by

$$v_e = (\beta+1)i_b R_E$$

That is,

$$v_e = [(\beta+1)R_E]i_b \qquad (3)$$

Dividing Eq. (3) by Eq. (2) yields

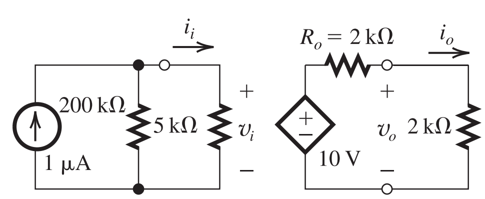
$$\frac{v_e}{v_b} = \frac{(\beta+1)R_E}{(\beta+1)R_E + r_\pi}$$

Dividing the numerator and denominator by $(\beta+1)$ gives

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi/(\beta+1)]} \qquad \text{Q.E.D}$$

**1.67** $R_o = \dfrac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \dfrac{10\ \text{V}}{5\ \text{mA}}$
$= 2\ \text{k}\Omega$

$$v_o = 10 \times \frac{2}{2+2} = 5\ \text{V}$$



$$A_v = \frac{v_o}{v_i} = \frac{10(2/4)}{1 \times 10^{-6} \times (200 \parallel 5) \times 10^3}$$

1025 V/V or 60.2 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{v_i/R_i}$$

$$= \frac{v_o}{v_i}\frac{R_i}{R_L} = 1025 \times \frac{5\ \text{k}\Omega}{2\ \text{k}\Omega}$$

= 2562.5 A/A or 62.8 dB

The overall current gain can be found as

$$\frac{i_o}{i_s} = \frac{v_o/R_L}{1\ \mu\text{A}} = \frac{5\ \text{V}/2\ \text{k}\Omega}{1\ \mu\text{A}}$$

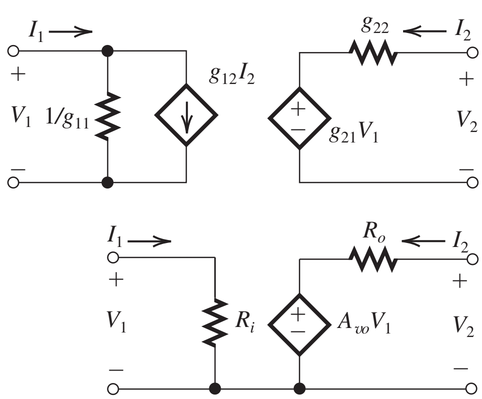
$$= \frac{2.5\ \text{mA}}{1\ \mu\text{A}} = 2500\ \text{A/A}$$

or 68 dB.

$$A_p = \frac{v_o^2/R_L}{i_i^2 R_i} = \frac{5^2/(2 \times 10^3)}{\left(10^{-6} \times \dfrac{200}{200+5}\right)^2 5 \times 10^3}$$

$= 2.63 \times 10^6$ W/W or 64.2 dB

**1.68**



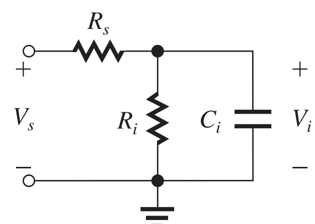

The correspondences between the current and voltage variables are indicated by comparing the two equivalent-circuit models above. At the outset we observe that at the input side of the $g$-parameter model, we have the controlled current source $g_{12}I_2$. This has no correspondence in the equivalent-circuit model of Fig. 1.16(a). It represents internal feedback, internal to the amplifier circuit. In developing the model of Fig. 1.16(a), we assumed that the amplifier is unilateral (i.e., has no internal feedback, or that the input side does not know what happens at the output side). If we neglect this internal feedback, that is, assume $g_{12} = 0$, we can compare the two models and thus obtain:

$$R_i = 1/g_{11}$$

$$A_{vo} = g_{21}$$

$$R_o = g_{22}$$

**1.69**




$$\frac{V_i}{V_s} = \frac{\dfrac{R_i\dfrac{1}{sC_i}}{R_i + \dfrac{1}{sC_i}}}{R_s + \left(\dfrac{R_i\dfrac{1}{sC_i}}{R_i + \dfrac{1}{sC_i}}\right)} = \frac{\dfrac{R_i}{1+sC_iR_i}}{R_s + \left(\dfrac{R_i}{1+sC_iR_i}\right)}$$

$$= \frac{R_i}{R_s + sC_iR_iR_s + R_i}$$

$$\frac{V_i}{V_s} = \frac{R_i}{(R_s+R_i) + sC_iR_iR_s} = \frac{\dfrac{R_i}{(R_s+R_i)}}{1 + s\left(\dfrac{C_iR_iR_s}{R_s+R_i}\right)}$$

which is a low-pass STC function with

$K = \dfrac{R_i}{R_s+R_i}$ and $\omega_0 = 1/[C_i(R_i \parallel R_s)]$.

For $R_s = 10\ \text{k}\Omega, R_i = 40\ \text{k}\Omega$, and $C_i = 5$ pF,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times (40 \parallel 10) \times 10^3} = 25\ \text{Mrad/s}$$

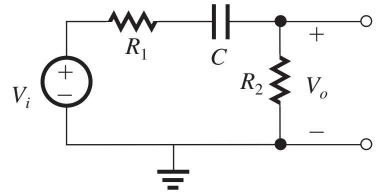
$$f_0 = \frac{25}{2\pi} = 4\ \text{MHz}$$

The dc gain is

$$K = \frac{40}{10+40} = 0.8\text{V/V}$$

**1.70** Using the voltage-divider rule.




$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \dfrac{1}{sC}}$$

$$T(s) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{s}{s + \dfrac{1}{C(R_1 + R_2)}}\right)$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2} \qquad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

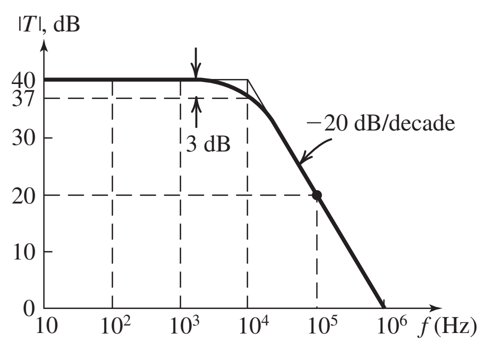
As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function, see that as $s \Rightarrow 0$, $T(s) \Rightarrow 0$; and as $s \rightarrow \infty$, $T(s) = R_2/(R_1 + R_2)$. Also, from the circuit, observe as $s \rightarrow \infty$, $(1/sC) \rightarrow 0$ and $V_o/V_i = R_2/(R_1 + R_2)$. Now, for $R_1 = 10$ kΩ, $R_2 = 40$ kΩ and $C = 1$ µF,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 1 \times 10^{-6}(10 + 40) \times 10^3}$$

$$= 3.18 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10 + 40}\frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

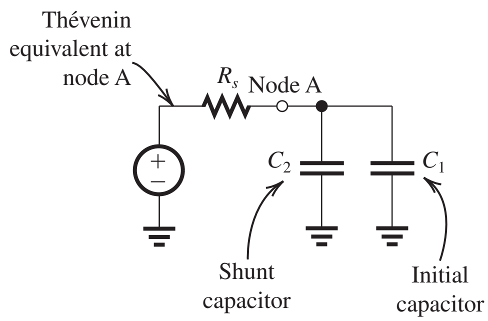
**1.71** The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of $10^4$ Hz. From our knowledge of the Bode plots for low-pass STC networks [Fig. 1.23(a)], we can complete the table entries and sketch the amplifier frequency response.




| $f$ (**Hz**) | $\|T\|$(**dB**) | $\angle T$(°) |
|---|---|---|
| 0 | 40 | 0 |
| 100 | 40 | 0 |
| 1000 | 40 | 0 |
| $10^4$ | 37 | −45° |
| $10^5$ | 20 | −90° |
| $10^6$ | 0 | −90° |

**1.72** $R_s = 100$ kΩ, since the 3-dB frequency is reduced by a very high factor (from 5 MHz to 100 kHz) $C_2$ must be much larger than $C_1$. Thus, neglecting $C_1$ we find $C_2$ from

$$100 \text{ kHz} \simeq \frac{1}{2\pi C_2 R_s}$$




$$= \frac{1}{2\pi C_2 \times 10^5}$$

$$\Rightarrow C_2 = 15.9 \text{ pF}$$

If the original 3-dB frequency (5 MHz) is attributable to $C_1$, then

$$5 \text{ MHz} = \frac{1}{2\pi C_1 R_s}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 5 \times 10^6 \times 10^5}$$

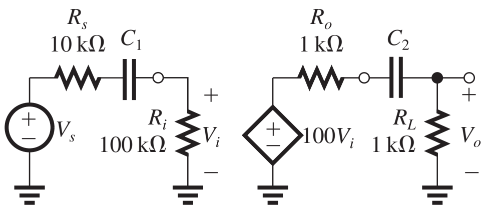
$$= 0.32 \text{ pF}$$

**1.73** For the input circuit, the corner frequency $f_{01}$ is found from

$$f_{01} = \frac{1}{2\pi C_1 (R_s + R_i)}$$

For $f_{01} \leq 100$ Hz,

$$\frac{1}{2\pi C_1 (10 + 100) \times 10^3} \leq 100$$




$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 1.4 \times 10^{-8}\ \text{F}$$

Thus we select $C_1 = 1 \times 10^{-7}$ F $= 0.1\ \mu$F. The actual corner frequency resulting from $C_1$ will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5\ \text{Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2(R_o + R_L)}$$

For $f_{02} \leq 100$ Hz,

$$\frac{1}{2\pi C_2(1+1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

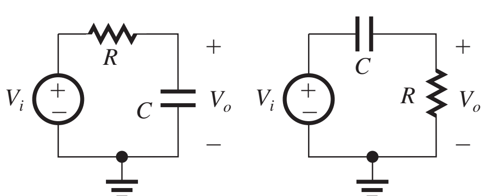
Select $C_2 = 1 \times 10^{-6} = 1\ \mu$F.

This will place the corner frequency at

$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80\ \text{Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right)\left(1 + \frac{s}{2\pi f_{02}}\right)}$$

**1.74** Circuits of Fig. 1.22:




For (a) $V_o = V_i\left(\frac{1/sC}{1/sC + R}\right)$

$$\frac{V_o}{V_i} = \frac{1}{1 + sCR}$$

which is of the form shown for the low-pass function in Table 1.2 with $K = 1$ and $\omega_0 = 1/RC$.

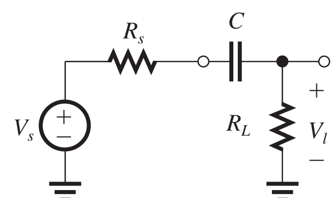
For (b) $V_o = V_i\left(\frac{R}{R + \frac{1}{sC}}\right)$

$$\frac{V_o}{V_i} = \frac{sRC}{1 + sCR}$$

$$\frac{V_o}{V_i} = \frac{s}{s + \frac{1}{RC}}$$

which is of the form shown in Table 1.2 for the high-pass function, with $K = 1$ and $\omega_0 = 1/RC$.

**1.75** Using the voltage divider rule,




$$\frac{V_l}{V_s} = \frac{R_L}{R_L + R_s + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_s} \frac{s}{s + \frac{1}{C(R_L + R_s)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_s} \quad \omega_0 = \frac{1}{C(R_L + R_s)}$$

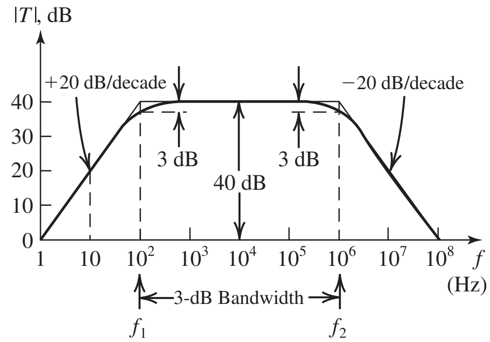
For $f_0 \leq 100$ Hz

$$\frac{1}{2\pi C(R_L + R_s)} \leq 100$$

$$\Rightarrow C \geq \frac{1}{2\pi \times 100(20 + 5) \times 10^3}$$

Thus, the smallest value of $C$ that will do the job is $C = 0.064\ \mu$F or 64 nF.

**1.76** From our knowledge of the Bode plots of STC low-pass and high-pass networks, we see that this amplifier has a midband gain of 40 dB, a low-frequency response of the high-pass STC type with $f_{3dB} = 10^2$ Hz, and a high-frequency response of the low-pass STC type with $f_{3dB} = 10^6$ Hz. We thus can sketch the amplifier frequency response and complete the table entries as follows.




| $f$(Hz) | 1 | 10 | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ |
|---|---|---|---|---|---|---|---|---|---|
| $\|T\|$(dB) | 0 | 20 | 37 | 40 | 40 | 40 | 37 | 20 | 0 |

**1.77** Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects, since the buffer amplifiers have infinite input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{ dB}}}{\omega_0}\right)^2}} = -1$$

$$\Rightarrow 1 + \left(\frac{\omega_{1\text{ dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\omega_0$$

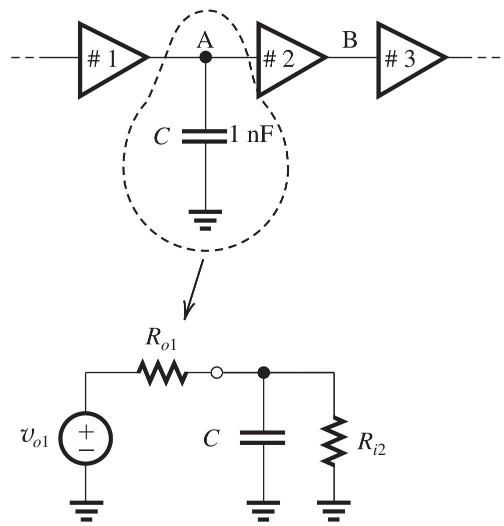
$$\omega_{1\text{dB}} = 0.51/CR$$

**1.78** Since when $C$ is connected to node A the 3-dB frequency is reduced by a large factor, the value of $C$ must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that $C$ is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$200\text{ kHz} = \frac{1}{2\pi C(R_{o1} \parallel R_{i2})}$$

$$\Rightarrow (R_{o1} \parallel R_{i2}) = \frac{1}{2\pi \times 200 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 0.8\text{ k}\Omega$$




Now $R_{i2} = 100$ kΩ.

Thus $R_{o1} \simeq 0.8$ kΩ

Similarly, for node B,

$$20\text{ kHz} = \frac{1}{2\pi C(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow R_{o2} \parallel R_{i3} = \frac{1}{2\pi \times 20 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 7.96\text{ k}\Omega$$

$$R_{o2} = 8.65\text{ k}\Omega$$

The designer should connect a capacitor of value $C_p$ to node B where $C_p$ can be found from

$$10\text{ kHz} = \frac{1}{2\pi C_p(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 7.96 \times 10^3}$$

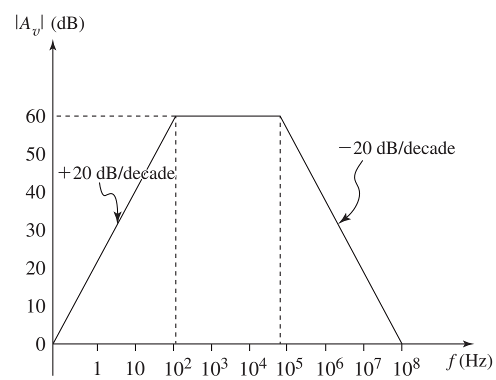
$$= 2\text{ nF}$$

Note that if she chooses to use node A, she would need to connect a capacitor 10 times larger!

**1.79** The LP factor $1/(1 + jf/10^5)$ results in a Bode plot like that in Fig. 1.23(a) with the 3-dB frequency $f_0 = 10^5$ Hz. The high-pass factor $1/(1 + 10^2/jf)$ results in a Bode plot like that in Fig. 1.24(a) with the 3-dB frequency

$f_0 = 10^2$ Hz.

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then shifting the resulting plot vertically by 60 dB (corresponding to the factor 1000 in the numerator). The result is as follows:




| $f =$ | 10 | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ | (Hz) |
|---|---|---|---|---|---|---|---|---|---|
| $\lvert A_v \rvert \simeq$ | 40 | 60 | 60 | 60 | 60 | 40 | 20 | 0 | (dB) |
| | | 57 | | | 57 | | | | |

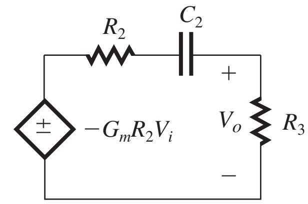
Bandwidth $= 10^5 - 10^2 = 99{,}900$ Hz

**1.80** $T_i(s) = \dfrac{V_i(s)}{V_s(s)} = \dfrac{1/sC_1}{1/sC_1 + R_1} = \dfrac{1}{sC_1R_1 + 1}$

LP with a 3-dB frequency

$$f_{0i} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^5} = 159\text{ kHz}$$

For $T_o(s)$, the following equivalent circuit can be used:




$$T_o(s) = \frac{V_o}{V_i} = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

$$= -G_m(R_2 \parallel R_3)\frac{s}{s + \dfrac{1}{C_2(R_2 + R_3)}}$$

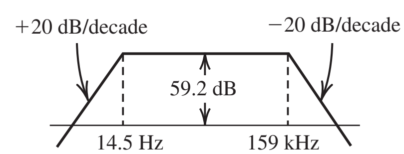
which is an HP, with

$$3\text{-dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 110 \times 10^3} = 14.5\text{ Hz}$$

$$\therefore T(s) = T_i(s)T_o(s)$$

$$= \frac{1}{1 + \dfrac{s}{2\pi \times 159 \times 10^3}} \times -909.1 \times \frac{s}{s + (2\pi \times 14.5)}$$




Bandwidth = 159 kHz – 14.5 Hz ≃ 159 kHz

**1.81** $V_i = V_s \dfrac{R_i}{R_s + R_i}$ (1)

(a) To satisfy constraint (1), namely,

$$V_i \geq \left(1 - \frac{x}{100}\right)V_s$$

we substitute in Eq. (1) to obtain

$$\frac{R_i}{R_s + R_i} \geq 1 - \frac{x}{100} \quad (2)$$

Thus

$$\frac{R_s + R_i}{R_i} \leq \frac{1}{1 - \dfrac{x}{100}}$$

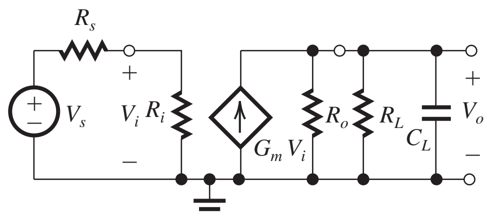
$$\frac{R_s}{R_i} \leq \frac{1}{1 - \dfrac{x}{100}} - 1 = \frac{\dfrac{x}{100}}{1 - \dfrac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_s} \geq \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in




$$R_i \geq R_s\left(\frac{100}{x} - 1\right) \qquad (3)$$

(b) The 3-dB frequency is determined by the parallel RC circuit at the output

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi}\frac{1}{C_L(R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_0}\right)$$

To obtain a value for $f_0$ greater than a specified value $f_{3dB}$ we select $R_o$ so that

$$\frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_o}\right) \geq f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \frac{1}{R_L}} \qquad (4)$$

(c) To satisfy constraint (c), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_s + R_i} G_m(R_o \parallel R_L)$$

For the dc gain to be greater than a specified value $A_0$,

$$\frac{R_i}{R_s + R_i} G_m(R_o \parallel R_L) \geq A_0$$

The first factor on the left-hand side is (from constraint (2)) greater or equal to $(1 - x/100)$. Thus

$$G_m \geq \frac{A_0}{\left(1 - \frac{x}{100}\right)(R_o \parallel R_L)} \qquad (5)$$

Substituting $R_s = 10$ kΩ and $x = 10\%$ in (3) results in

$$R_i \geq 10\left(\frac{100}{100} - 1\right) = 90 \text{ k}\Omega$$

Substituting $f_{3dB} = 2$ MHz, $C_L = 20$ pF, and $R_L = 10$ kΩ in Eq. (4) results in

$$R_o \leq \frac{1}{2\pi \times 2 \times 10^6 \times 20 \times 10^{-12} - \frac{1}{10^4}}$$

$= 6.61$ kΩ

Substituting $A_0 = 100$, $x = 10\%$, $R_L = 10$ kΩ, and $R_o = 6.61$ kΩ, Eq. (5) results in

$$G_m \geq \frac{100}{\left(1 - \frac{10}{100}\right)(10 \parallel 6.61) \times 10^3} = 27.9 \text{ mA/V}$$

**1.82** Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \text{ and } Z_2 = R_2 \parallel \frac{1}{sC_2}$$

It is obviously more convenient to work in terms of admittances. Therefore we express $V_o/V_i$ in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute $Y_1 = (1/R_1) + sC_1$ and $Y_2 = (1/R_2) + sC_2$ to obtain

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 + C_2}\frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

This transfer function will be independent of frequency ($s$) if the second factor reduces to unity.

This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

which can be simplified as follows:

$$\frac{C_1 + C_2}{C_1} = R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1R_1 = C_2R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1), can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ($C_1R_1 = C_2R_2$), its transmission can be determined either by its two resistors $R_1$, $R_2$ or by its two capacitors. $C_1$, $C_2$, and the transmission is *not* a function of frequency.

**1.83** The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of 5.7° at $f = 100$ Hz. Using the equation for $\angle T(j\omega)$ from Table 1.2, we obtain

$$\tan^{-1} \frac{f_0}{100} = 5.7^\circ \Rightarrow f_0 = 10 \text{ Hz}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of −5.7° at $f = 1$ kHz. Using the relationship for $\angle T(j\omega)$ given in Table 1.2, we obtain for the LP STC circuit.

$$-\tan^{-1} \frac{10^3}{f_0} = -5.7^\circ \Rightarrow f_0 \simeq 10 \text{ kHz}$$

At $f = 100$ Hz, the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{10}{100}\right)^2}} = -0.04 \text{ dB}$$

Similarly, at the drop in gain $f = 1$ kHz is caused by the LP STC network. The drop in gain is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{1000}{10,000}\right)^2}} = -0.04 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at $f = 10$ Hz and $f = 10$ kHz.

**1.84** Use the expression in Eq. (1.26), with

$B = 7.3 \times 10^{15} \text{ cm}^{-3}\text{K}^{-3/2}$;

$k = 8.62 \times 10^{-5} \text{eV/K}$; and $E_g = 1.12$ V

we have

$T = -55°\text{C} = 218$ K:

$$n_i = 2.68 \times 10^6 \text{ cm}^{-3}; \ \frac{N}{n_i} = 1.9 \times 10^{16}$$

That is, one out of every $1.9 \times 10^{16}$ silicon atoms is ionized at this temperature.

$T = 0°\text{C} = 273$ K:

$$n_i = 1.52 \times 10^9 \text{ cm}^{-3}; \ \frac{N}{n_i} = 3.3 \times 10^{13}$$

$T = 20°\text{C} = 293$ K:

$$n_i = 8.60 \times 10^9 \text{ cm}^{-3}; \ \frac{N}{n_i} = 5.8 \times 10^{12}$$

$T = 75°\text{C} = 348$ K:

$$n_i = 3.70 \times 10^{11} \text{ cm}^{-3}; \ \frac{N}{n_i} = 1.4 \times 10^{11}$$

$T = 125°\text{C} = 398$ K:

$$n_i = 4.72 \times 10^{12} \text{ cm}^{-3}; \ \frac{N}{n_i} = 1.1 \times 10^{10}$$

**1.85** Use Eq. (1.26) to find $n_i$,

$$n_i = BT^{3/2} \, e^{-E_g/2kT}$$

Substituting the values given in the problem,

$$n_i = 3.56 \times 10^{14} (300)^{3/2} e^{-1.42/(2 \times 8.62 \times 10^{-5} \times 300)}$$

$$= 2.2 \times 10^6 \text{ carriers/cm}^3$$

**1.86** The concentration of free carriers (both electrons and holes) in intrinsic silicon is found in Example 3.1 to be $1.5 \times 10^{10}$ carriers/cm$^3$ at room temperature. Multiplying this by the volume of the wafer gives

$$1.5 \times 10^{10} \times \frac{\pi \times 15^2 \times 0.3}{4} =$$

$$7.95 \times 10^{10} \text{ free electrons}$$

**1.87** Since $N_A \gg n_i$, we can write

$p_p \approx N_A = 5 \times 10^{18}\ \text{cm}^{-3}$

Using Eq. (1.27), we have

$$n_p = \frac{n_i^2}{p_p} = 45\ \text{cm}^{-3}$$

**1.88** Hole concentration in intrinsic Si $= n_i$

$n_i = BT^{3/2}\ e^{-E_g/2kT}$

$= 7.3 \times 10^{15}(300)^{3/2}e^{-1.12/(2\times 8.62\times 10^{-5}\times 300)}$

$= 1.5 \times 10^{10}$ holes/cm$^3$

In phosphorus-doped Si, hole concentration drops below the intrinsic level by a factor of $10^8$.

$\therefore$ Hole concentration in P-doped Si is

$$p_n = \frac{1.5 \times 10^{10}}{10^8} = 1.5 \times 10^2\ \text{cm}^{-3}$$

Now, $n_n \simeq N_D$ and $p_n n_n = n_i^2$

$$n_n = n_i^2/p_n = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^2}$$

$= 1.5 \times 10^{18}\ \text{cm}^{-3}$

$N_D = n_n = 1.5 \times 10^{18}$ atoms/cm$^3$

**1.89** $T = 27°\text{C} = 273 + 27 = 300$ K

At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

Phosphorus-doped Si:

$n_n \simeq N_D = 10^{17}/\text{cm}^3$

$$p_n = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3/\text{cm}^3$$

Hole concentration $= p_n = 2.25 \times 10^3/\text{cm}^3$

$T = 125°\text{C} = 273 + 125 = 398$ K

At 398 K, $n_i = BT^{3/2}e^{-E_g/2kT}$

$= 7.3 \times 10^{15} \times (398)^{3/2}e^{-1.12/(2\times 8.62\times 10^{-5}\times 398)}$

$= 4.72 \times 10^{12}/\text{cm}^3$

$$p_n = \frac{n_i^2}{N_D} = 2.23 \times 10^8/\text{cm}^3$$

At 398 K, hole concentration is

$p_n = 2.23 \times 10^8/\text{cm}^3$

**1.90** (a) The resistivity of silicon is given by Eq. (1.41).

For intrinsic silicon,

$p = n = n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$

Using $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$ and

$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$, and $q = 1.6 \times 10^{-19}$ C we have

$\rho = 2.28 \times 10^5\ \Omega$-cm.

Using $R = \rho \cdot \dfrac{L}{A}$ with $L = 0.001$ cm and

$A = 3 \times 10^{-8}\ \text{cm}^2$, we have

$R = 7.6 \times 10^9\ \Omega$.

(b) $n_n \approx N_D = 5 \times 10^{16}\ \text{cm}^{-3}$;

$$p_n = \frac{n_i^2}{n_n} = 4.5 \times 10^3\ \text{cm}^{-3}$$

Using $\mu_n = 1200\ \text{cm}^2/\text{V}\cdot\text{s}$ and

$\mu_p = 400\ \text{cm}^2/\text{V}\cdot\text{s}$, we have

$\rho = 0.10\ \Omega$-cm; $R = 3.33\ \text{k}\Omega$.

(c) $n_n \approx N_D = 5 \times 10^{18}\ \text{cm}^{-3}$;

$$p_n = \frac{n_i^2}{n_n} = 45\ \text{cm}^{-3}$$

Using $\mu_n = 1200\ \text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 400\ \text{cm}^2/\text{V}\cdot\text{s}$, we have

$\rho = 1.0 \times 10^{-3}\ \Omega$-cm; $R = 33.3\ \Omega$.

As expected, since $N_D$ is increased by 100, the resistivity decreases by the same factor.

(d) $p_p \approx N_A = 5 \times 10^{16}\ \text{cm}^{-3}$; $n_p = \dfrac{n_i^2}{n_n}$

$= 4.5 \times 10^3\ \text{cm}^{-3}$

$\rho = 0.31\ \Omega$-cm; $R = 10.42\ \text{k}\Omega$

(e) Since $\rho$ is given to be $2.8 \times 10^{-6}\ \Omega$-cm, we directly calculate $R = 9.33 \times 10^{-2}\ \Omega$.

**1.91** Cross-sectional area of Si bar

$= 5 \times 4 = 20\ \mu\text{m}^2$

Since $1\ \mu\text{m} = 10^{-4}$ cm, we get

$= 20 \times 10^{-8}\ \text{cm}^2$

Current $I = Aq(p\mu_p + n\mu_n)E$

$= 20 \times 10^{-8} \times 1.6 \times 10^{-19}$

$$(10^{16} \times 500 + 10^4 \times 1200) \times \frac{1\ \text{V}}{10 \times 10^{-4}}$$

$= 160\ \mu\text{A}$

**1.92** Use Eq. (1.45): $\dfrac{D_n}{\mu_n} = \dfrac{D_p}{\mu_p} = V_T$

$D_n = \mu_n V_T$ and $D_p = \mu_p V_T$ where $V_T = 25.9$ mV.

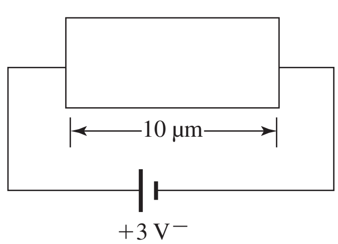
| **Doping Concentration (carriers/cm$^3$)** | **$\mu_n$ cm$^2$/V · s** | **$\mu_p$ cm$^2$/V · s** | **$D_n$ cm$^2$/s** | **$D_p$ cm$^2$/s** |
|---|---|---|---|---|
| Intrinsic | 1350 | 480 | 35 | 12.4 |
| $10^{16}$ | 1200 | 400 | 31 | 10.4 |
| $10^{17}$ | 750 | 260 | 19.4 | 6.7 |
| $10^{18}$ | 380 | 160 | 9.8 | 4.1 |

**1.93** Electric field:

$$E = \frac{3\ \text{V}}{10\ \mu\text{m}} = \frac{3\ \text{V}}{10 \times 10^{-6}\ \text{m}}$$

$$= \frac{3\ \text{V}}{10 \times 10^{-4}\ \text{cm}}$$

$= 3000$ V/cm




$v_{p\text{-drift}} = \mu_p E = 480 \times 3000$

$= 1.44 \times 10^6$ cm/s

$v_{n\text{-drift}} = \mu_n E = 1350 \times 3000$

$= 4.05 \times 10^6$ cm/s

$$\frac{v_n}{v_p} = \frac{4.05 \times 10^6}{1.44 \times 10^6} = 2.8125 \quad \text{or}$$

$v_n = 2.8125\ v_p$

Or, alternatively, it can be shown as

$$\frac{v_n}{v_p} = \frac{\mu_n E}{\mu_p E} = \frac{\mu_n}{\mu_p} = \frac{1350}{480}$$

$= 2.8125$

**1.94** $J_{\text{drift}} = q(n\mu_n + p\mu_p)E$

Here $n = N_D$, and since it is $n$-type silicon, one can assume $p \ll n$ and ignore the term $p\mu_p$. Also,

$$E = \frac{1\ \text{V}}{10\ \mu\text{m}} = \frac{1\ \text{V}}{10 \times 10^{-4}\ \text{cm}} = 10^3\ \text{V/cm}$$

Need $J_{\text{drift}} = 2\ \text{mA}/\mu\text{m}^2 = qN_D\mu_n E$

$$\frac{2 \times 10^{-3}\ \text{A}}{10^{-8}\ \text{cm}^2} = 1.6 \times 10^{-19}\ N_D \times 1350 \times 10^3$$

$\Rightarrow N_D = 9.26 \times 10^{17}/\text{cm}^3$

**1.95**

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

From Fig. P1.95,

$$\frac{dp}{dx} = -\frac{10^8 p_{n0} - p_{n0}}{W} \simeq -\frac{10^8 p_{n0}}{50 \times 10^{-7}}$$

since $1\ \text{nm} = 10^{-7}$ cm

$$\frac{dp}{dx} = -\frac{10^8 \times 2.25 \times 10^4}{50 \times 10^{-7}}$$

$= -4.5 \times 10^{17}$

Hence

$$J_p = -qD_p \frac{dp}{dx}$$

$= -1.6 \times 10^{-19} \times 12 \times (-4.5 \times 10^{17})$

$= 0.864\ \text{A/cm}^2$

**1.96** From Table 1.3,

$V_T$ at 300 K = 25.9 mV

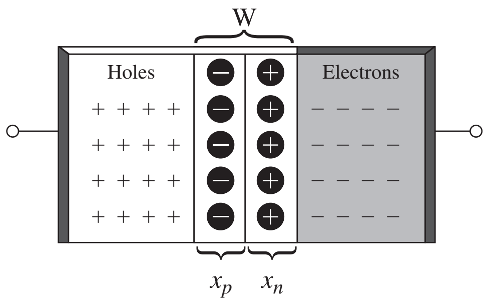
Using Eq. (1.46), built-in voltage $V_0$ is obtained:

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 25.9 \times 10^{-3} \times$$

$$\ln\left(\frac{10^{17} \times 10^{16}}{\left(1.5 \times 10^{10}\right)^2}\right)$$

$= 0.754$ V




Depletion width

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0} \leftarrow \text{Eq. (1.50)}$$

$W =$

$$\sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{17}} + \frac{1}{10^{16}}\right) \times 0.754}$$

$= 0.328 \times 10^{-4}$ cm $= 0.328$ μm

Use Eqs. (1.51) and (1.52) to find $x_n$ and $x_p$:

$$x_n = W\frac{N_A}{N_A + N_D} = 0.328 \times \frac{10^{17}}{10^{17} + 10^{16}}$$

$= 0.298$ μm

$$x_p = W\frac{N_D}{N_A + N_D} = 0.328 \times \frac{10^{16}}{10^{17} + 10^{16}}$$

$= 0.03$ μm

Use Eq. (1.53) to calculate charge stored on either side:

$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W$, where junction area

$= 100\ \mu\text{m}^2 = 100 \times 10^{-8}\ \text{cm}^2$

$$Q_J = 100 \times 10^{-8} \times 1.6 \times 10^{-19}\left(\frac{10^{17} \cdot 10^{16}}{10^{17} + 10^{16}}\right)$$

$$\times\ 0.328 \times 10^{-4}$$

Hence, $Q_J = 4.8 \times 10^{-14}$ C

**1.97** Equation (1.49):

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0},$$

Since $N_A \gg N_D$, we have

$$W \simeq \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}V_0}$$

$$V_0 = \frac{qN_D}{2\epsilon_s} \cdot W^2$$

Here $W = 0.2\ \mu\text{m} = 0.2 \times 10^{-4}$ cm

$$\text{So } V_0 = \frac{1.6 \times 10^{-19} \times 10^{16} \times \left(0.2 \times 10^{-4}\right)^2}{2 \times 1.04 \times 10^{-12}}$$

$= 0.31$ V

$$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W \cong AqN_D W$$

since $N_A \gg N_D$, we have $Q_J = 3.2$ fC.

**1.98** Using Eq. (1.46) and $N_A = N_D$

$= 5 \times 10^{16}\ \text{cm}^{-3}$ and $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$,

we have $V_0 = 778$ mV.

Using Eq. (1.50) and
$\epsilon_s = 11.7 \times 8.854 \times 10^{-14}$ F/cm, we have
$W = 2 \times 10^{-5}$ cm $= 0.2$ μm. The extension of the depletion width into the $n$ and $p$ regions is given in Eqs. (1.51) and (1.52), respectively:

$$x_n = W \cdot \frac{N_A}{N_A + N_D} = 0.1\ \mu\text{m}$$

$$x_p = W \cdot \frac{N_D}{N_A + N_D} = 0.1\ \mu\text{m}$$

Since both regions are doped equally, the depletion region is symmetric.

Using Eq. (1.53) and
$A = 20\ \mu\text{m}^2 = 20 \times 10^{-8}\ \text{cm}^2$, the charge magnitude on each side of the junction is

$Q_J = 1.6 \times 10^{-14}$ C.

**1.99** Using Eq. (1.47) or (1.48), we have charge stored: $Q_J = qAx_nN_D$.

Here $x_n = 0.1\ \mu\text{m} = 0.1 \times 10^{-4}$ cm

$A = 10\ \mu\text{m} \times 10\ \mu\text{m} = 10 \times 10^{-4}$ cm

$\times\ 10 \times 10^{-4}$ cm

$= 100 \times 10^{-8}\ \text{cm}^2$

So,

$Q_J = 1.6 \times 10^{-19} \times 100 \times 10^{-8} \times 0.1 \times 10^{-4} \times 10^{18}$

$= 1.6$ pC

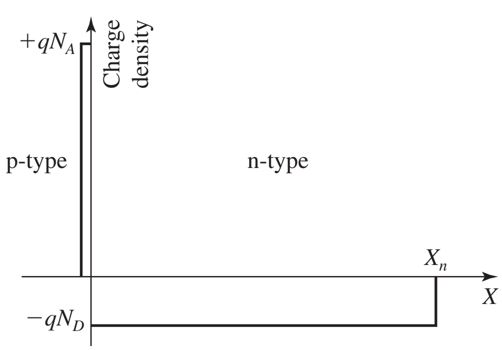
**1.100** $V_0 = V_T \ln\left(\dfrac{N_A N_D}{n_i^2}\right)$

If $N_A$ or $N_D$ is increased by a factor of 10, then new value of $V_0$ will be

$$V_0 = V_T \ln\left(\frac{10\ N_A N_D}{n_i^2}\right)$$

The change in the value of $V_0$ is $V_T \ln 10 = 59.6$ mV.

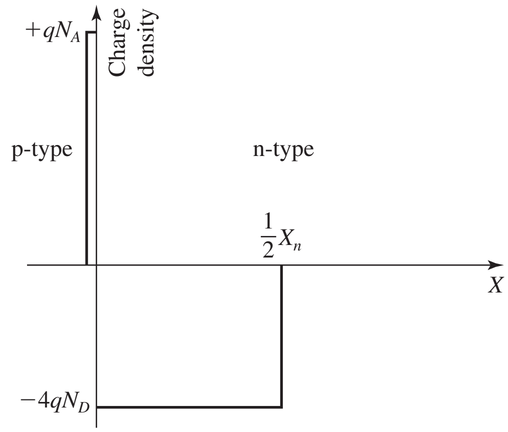
**1.101** This is a one-sided junction, with the depletion layer extending almost entirely into the more lightly doped ($n$-type) material. A thin space-charge region in the $p$-type region stores the same total charge with higher much higher charge density.




Increasing $N_D$ by 4× will increase the charge density on the $n-type$ side of the junction by 4×. We see from Eq. (1.45) that since $N_A \gg N_D \gg n_i$ (i.e. at least a couple of orders of magnitude) a 4× increase in $N_D$ will have comparatively little effect on $V_0$. Thus, we can assume $V_0$ is unchanged in Eq. (1.49), and the junction width (residing almost entirely in the $n$-type material) is

$$W \approx \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_A}V_0} \propto \frac{1}{\sqrt{N_D}}$$

A 4× increase in $N_D$ therefore results in a 2× decrease in the width of the depletion region on the $n$-type side of the junction, as illustrated below.




**1.102** The area under the triangle is equal to the built-in voltage.

$$V_0 = \frac{1}{2}E_{\max}W$$

Using Eq. (1.49):

$$V_0 = \frac{E_{\max}}{2}\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)} \times \sqrt{V_0}$$

$$\Rightarrow E_{\max} = \sqrt{\frac{2qN_AN_D}{\epsilon_s(N_A+N_D)}V_0}$$

Substituting Eq. (1.45) for $V_0$,

$$E_{\max} = \sqrt{\frac{2kTN_AN_D}{\epsilon_s(N_A+N_D)}\ln\left(\frac{N_AN_D}{n_i^2}\right)}$$

Finally, we can substitute for $n_i$ using Eq. (1.26)

$$E_{\max} = \sqrt{\frac{2kTN_AN_D}{\epsilon_s(N_A+N_D)}\left[\ln(N_AN_D) - 2\ln B - 3\ln T + \frac{E_g}{kT}\right]}$$

**1.103** Using Eq. (1.46) with $N_A = 10^{17}\ \text{cm}^{-3}$, $N_D = 10^{16}\ \text{cm}^{-3}$, and $n_i = 1.5 \times 10^{10}$, we have $V_0 = 754$ mV

Using Eq. (1.55) with $V_R = 5$ V, we have $W = 0.907\ \mu$m.

Using Eq. (1.56) with $A = 1 \times 10^{-6}\ \text{cm}^2$, we have $Q_J = 13.2 \times 10^{-14}$ C.

**1.104** Equation (1.65):

$$I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

$A = 100\ \mu\text{m}^2 = 100 \times 10^{-8}\ \text{cm}^2$

$I_S = 100 \times 10^{-8} \times 1.6 \times 10^{-19} \times \left(1.5 \times 10^{10}\right)^2$

$$\left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{17}}\right)$$

$= 7.85 \times 10^{-17}$ A

$I \cong I_S e^{V/V_T}$

$= 7.85 \times 10^{-17} \times e^{750/25.9}$

$\cong 0.3$ mA

**1.105** Equation (1.54):

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0\left(1 + \frac{V_R}{V_0}\right)}$$

$$= W_0\sqrt{1 + \frac{V_R}{V_0}}$$

Equation (1.55):

$$Q_j = A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) \cdot (V_0 + V_R)}$$

$$= A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) V_0 \cdot \left(1 + \frac{V_R}{V_0}\right)}$$

$$= Q_{J0}\sqrt{1 + \frac{V_R}{V_0}}$$

**1.106** Equation (1.62):

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)\left(e^{V/V_T} - 1\right)$$

Here $I_p = Aqn_i^2 \dfrac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$

$$I_n = Aqn_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T} - 1\right)$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n} \cdot \frac{L_n}{L_p} \cdot \frac{N_A}{N_D}$$

$$= \frac{10}{20} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}}$$

$$\frac{I_p}{I_n} = 100$$

Now $I = I_p + I_n = 100\, I_n + I_n \equiv 1$ mA

$I_n = \dfrac{1}{101}$ mA $= 0.0099$ mA

$I_p = 1 - I_n = 0.9901$ mA

**1.107** $n_i = BT^{3/2}e^{-E_g/2kT}$

At 300 K,

$n_i = 7.3 \times 10^{15} \times (300)^{3/2} \times e^{-1.12/\left(2 \times 8.62 \times 10^{-5} \times 300\right)}$

$= 1.4939 \times 10^{10}/\text{cm}^2$

$n_i^2$ (at 300 K) $= 2.232 \times 10^{20}$

At 305 K,

$n_i = 7.3 \times 10^{15} \times (305)^{3/2} \times e^{-1.12/\left(2 \times 8.62 \times 10^{-5} \times 305\right)}$

$= 2.152 \times 10^{10}$

$n_i^2$ (at 305 K) $= 4.631 \times 10^{20}$

so $\dfrac{n_i^2 \text{ (at 305 K)}}{n_i^2 \text{ (at 300 K)}} = 2.152$

Thus $I_S$ approximately doubles for every 5°C rise in temperature.

**1.108** Equation (1.63)

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)\left(e^{V/V_T} - 1\right)$$

So $I_p = Aqn_i^2 \dfrac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$

$$I_n = Aqn_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T} - 1\right)$$

For $p^+\!-\!n$ junction $N_A \gg N_D$, thus $I_p \gg I_n$ and

$$I \simeq I_p = Aqn_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$$

For this case using Eq. (1.65):

$$I_S \simeq Aqn_i^2 \frac{D_p}{L_p N_D} = 10^4 \times 10^{-8} \times 1.6 \times 10^{-19}$$

$$\times\left(1.5 \times 10^{10}\right)^2 \frac{10}{10 \times 10^{-4} \times 10^{17}}$$

$= 3.6 \times 10^{-16}$ A

$I = I_S\left(e^{V/V_T} - 1\right) = 1.0 \times 10^{-3}$

$$3.6 \times 10^{-16}\left(e^{V/(25.9\times10^{-3})} - 1\right) = 1.0 \times 10^{-3}$$

$$\Rightarrow V = 0.742 \text{ V}$$

**1.109** $V_Z = 12$ V

Rated power dissipation of diode = 0.25 W.

If continuous current "$I$" raises the power dissipation to half the rated value, then

$$12 \text{ V} \times I = \frac{1}{2} \times 0.25 \text{ W}$$

$$I = 10.42 \text{ mA}$$

Since breakdown occurs for only half the time, the breakdown current $I$ can be determined from

$$I \times 12 \times \frac{1}{2} = 0.25 \text{ W}$$

$$\Rightarrow I = 41.7 \text{ mA}$$

**1.110** Equation (1.73), $C_j = \dfrac{C_{j0}}{\left(1 + \dfrac{V_R}{V_0}\right)^m}$

For $V_R = 1$ V, $C_j = \dfrac{0.4 \text{ pF}}{\left(1 + \dfrac{1}{0.75}\right)^{1/3}}$

$$= 0.3 \text{ pF}$$

For $V_R = 10$ V, $C_j = \dfrac{0.4 \text{ pF}}{\left(1 + \dfrac{10}{0.75}\right)^{1/3}}$

$$= 0.16 \text{ pF}$$

**1.111** Equation (1.81):

$$C_d = \left(\frac{\tau_T}{V_T}\right)I$$

$$5 \text{ pF} = \left(\frac{\tau_T}{25.9 \times 10^{-3}}\right) \times 1 \times 10^{-3}$$

$$\tau_T = 5 \times 10^{-12} \times 25.9$$

$$= 129.5 \text{ ps}$$

For $I = 0.1$ mA:

$$C_d = \left(\frac{\tau_T}{V_T}\right) \times I$$

$$= \left(\frac{129.5 \times 10^{-12}}{25.9 \times 10^{-3}}\right) \times 0.1 \times 10^{-3} = 0.5 \text{ pF}$$

**1.112** Equation (1.72):

$$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 25.9 \times 10^{-3} \times \ln\left(\frac{10^{17} \times 10^{16}}{(1.5 \times 10^{10})^2}\right)$$

$$= 0.754 \text{ V}$$

$$C_{j0} = 100 \times 10^{-8}$$

$$\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)\left(\frac{10^{17} \times 10^{16}}{10^{17} + 10^{16}}\right)\frac{1}{0.754}}$$

$$= 31.6 \text{ fF}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \dfrac{V_R}{V_0}}} = \frac{31.6 \text{ fF}}{\sqrt{1 + \dfrac{3}{0.754}}}$$

$$= 14.16 \text{ fF}$$

**1.113** Equation (1.66):

$$\alpha = A\sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}$$

Equation (1.68):

$$C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}}$$

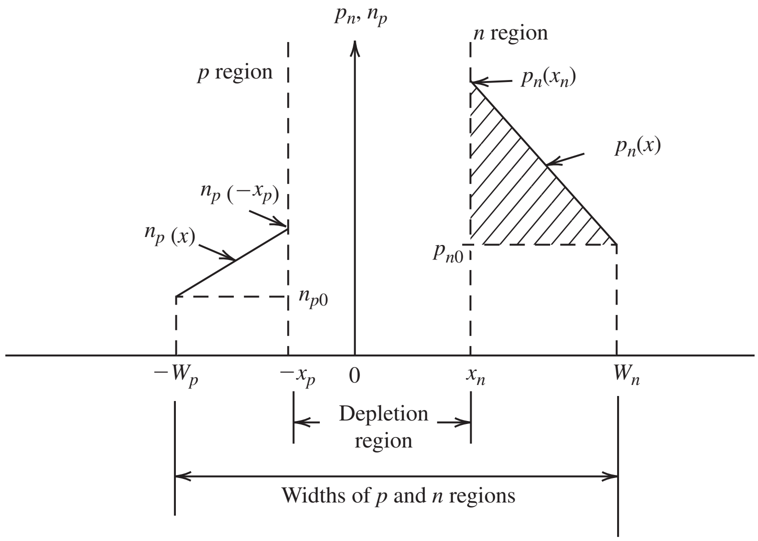
Substitute for $\alpha$ from Eq. (1.66):

$$C_j = \frac{A\sqrt{2\epsilon_s q \dfrac{N_A N_D}{N_A + N_D}}}{2\sqrt{V_0 + V_R}} \times \frac{\sqrt{\epsilon_s}}{\sqrt{\epsilon_s}}$$

$$= A\epsilon_s \times \frac{1}{\sqrt{\dfrac{2\epsilon_s}{q}\left(\dfrac{N_A + N_D}{N_A N_D}\right)(V_0 + V_R)}}$$

$$= \epsilon_s A \frac{1}{\sqrt{\dfrac{2\epsilon_s}{q}\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right)(V_0 + V_R)}}$$

$$= \frac{\epsilon_s A}{W}$$




**1.114** (a) See figure at top of page.

(b) The current $I = I_p + I_n$.

Find current component $I_p$:

$$p_n(x_n) = p_{n0}e^{V/V_T} \text{ and } p_{n0} = \frac{n_i^2}{N_D}$$

$$I_p = AJ_p = AqD_p\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{p_n(x_n) - p_{n0}}{W_n - x_n} = \frac{p_{n0}e^{V/V_T} - p_{n0}}{W_n - x_n}$$

$$= p_{n0}\frac{\left(e^{V/V_T} - 1\right)}{W_n - x_n}$$

$$= \frac{n_i^2}{N_D}\frac{\left(e^{V/V_T} - 1\right)}{(W_n - x_n)}$$

$$\therefore I_p = AqD_p\frac{dp}{dx}$$

$$= Aqn_i^2\frac{D_p}{(W_n - x_n)N_D} \times \left(e^{V/V_T} - 1\right)$$

Similarly,

$$I_n = Aqn_i^2\frac{D_n}{\left(W_p - x_p\right)N_A} \times \left(e^{V/V_T} - 1\right)$$

$$I = I_p + I_n$$

$$= Aqn_i^2\left[\frac{D_p}{(W_n - x_n)\,N_D} + \frac{D_n}{\left(W_p - x_p\right)N_A}\right]$$

$$\times \left(e^{V/V_T} - 1\right)$$

The excess change, $Q_p$, can be obtained by multiplying the area of the shaded triangle of the $p_n(x)$ distribution graph by $Aq$.

$$Q_p = Aq \times \frac{1}{2}\left[p_n\,(x_n) - p_{n0}\right](W_n - x_n)$$

$$= \frac{1}{2}Aq\left[p_{n0}e^{V/V_T} - p_{n0}\right](W_n - x_n)$$

$$= \frac{1}{2}\,Aqp_{n0}\left(e^{V/V_T} - 1\right)(W_n - x_n)$$

$$= \frac{1}{2}\,Aq\frac{{n_i}^2}{N_D}\,(W_n - x_n)\left(e^{V/V_T} - 1\right)$$

$$= \frac{1}{2}\frac{(W_n - x_n)^2}{D_p}\cdot I_p$$

$$\simeq \frac{1}{2}\frac{W_n^2}{D_P}\cdot I_p \text{ for } W_n \gg x_n$$

(c) For $Q \simeq Q_p$, $I \simeq I_p$,

$$Q \simeq \frac{1}{2}\frac{W_n^2}{D_p}I$$

Thus, $\tau_T = \dfrac{1}{2}\dfrac{W_n^2}{D_p}$, and

$$C_d = \frac{dQ}{dV} = \tau_T\frac{dI}{dV}$$

But $I = I_S\left(e^{V/V_T} - 1\right)$

$$\frac{dI}{dV} = \frac{I_S e^{V/V_T}}{V_T} \simeq \frac{I}{V_T}$$

so $C_d \cong \tau_T \cdot \dfrac{I}{V_T}$.

(d) $C_d = \dfrac{1}{2}\dfrac{W_n^2}{10}\dfrac{1 \times 10^{-3}}{25.9 \times 10^{-3}} = 8 \times 10^{-12}$ F

Solve for $W_n$:

$W_n = 0.64\ \mu$m

## Chapter 2

### *Solutions to Exercises within the Chapter*

**Ex: 2.1** The minimum number of terminals required by a single op amp is 5: two input terminals, one output terminal, one terminal for positive power supply, and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amps can all share one terminal for positive power supply and one terminal for negative power supply.

**Ex: 2.2** Relevant equations are:
$v_3 = A(v_2 - v_1)$; $v_{Id} = v_2 - v_1$,
$v_{Icm} = \frac{1}{2}(v_1 + v_2)$

(a)
$$v_1 = v_2 - \frac{v_3}{A} = 0 - \frac{4}{10^3} = -0.004 \text{ V} = -4 \text{ mV}$$
$$v_{Id} = v_2 - v_1 = 0 - (-0.004) = +0.004 \text{ V}$$
$$= 4 \text{ mV}$$
$$v_{Icm} = \frac{1}{2}(-4 \text{ mV} + 0) = -2 \text{ mV}$$

(b) $-10 = 10^3(2 - v_1) \Rightarrow v_1 = 2.01$ V
$$v_{Id} = v_2 - v_1 = 2 - 2.01 = -0.01 \text{ V} = -10 \text{ mV}$$
$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.01 + 2) = 2.005 \text{ V}$$
$$\simeq 2 \text{ V}$$

(c)
$$v_3 = A(v_2 - v_1) = 10^3(1.998 - 2.002) = -4 \text{ V}$$
$$v_{Id} = v_2 - v_1 = 1.998 - 2.002 = -4 \text{ mV}$$
$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(2.002 + 1.998) = 2 \text{ V}$$

(d)
$$-1.2 = 10^3[v_2 - (-1.2)] = 10^3(v_2 + 1.2)$$
$$\Rightarrow v_2 = -1.2012 \text{ V}$$
$$v_{Id} = v_2 - v_1 = -1.2012 - (-1.2)$$
$$= -0.0012 \text{ V} = -1.2 \text{ mV}$$
$$v_{Icm} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-1.2 + (-1.2012)]$$
$$\simeq -1.2 \text{ V}$$

**Ex: 2.3** From Fig. E2.3 we have: $v_3 = \mu v_d$ and

$$v_d = (G_m v_2 - G_m v_1)R = G_m R(v_2 - v_1)$$

Therefore:

$$v_3 = \mu G_m R(v_2 - v_1)$$

That is, the open-loop gain of the op amp is $A = \mu G_m R$. For $G_m = 20$ mA/V and $\mu = 50$, we have:

$A = 50 \times 20 \times 5 = 5000$ V/V, or equivalently, 74 dB.

**Ex: 2.4** The gain and input resistance of the inverting amplifier circuit shown in Fig. 2.5 are $-\frac{R_2}{R_1}$ and $R_1$, respectively. Therefore, we have:

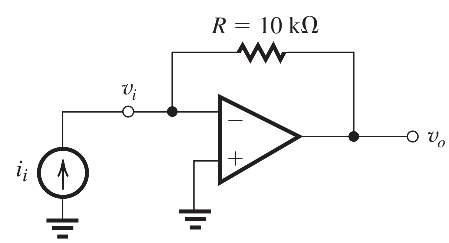
$R_1 = 100$ kΩ and

$$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10\, R_1$$

Thus:

$$R_2 = 10 \times 100 \text{ k}\Omega = 1 \text{ M}\Omega$$

**Ex: 2.5**




From Table 1.1 we have:

$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0}$ ; that is, output is open circuit

The negative input terminal of the op amp (i.e., $v_i$) is a virtual ground, thus $v_i = 0$:

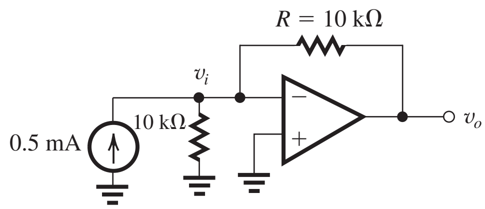
$$v_o = v_i - Ri_i = 0 - Ri_i = -Ri_i$$
$$R_m = \left.\frac{v_o}{i_i}\right|_{i_o=0} = -\frac{Ri_i}{i_i} = -R \Rightarrow R_m = -R$$
$$= -10 \text{ k}\Omega$$

$R_i = \frac{v_i}{i_i}$ and $v_i$ is a virtual ground ($v_i = 0$),

thus $R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0\ \Omega$

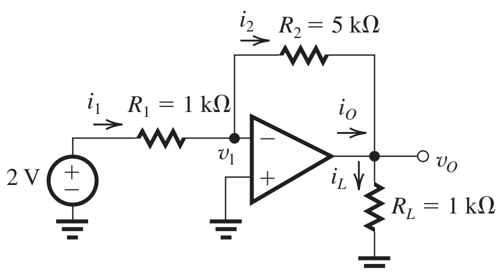
Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is $R_o = 0\ \Omega$.




Connecting the signal source shown in Fig. E2.5 to the input of this amplifier, we have:
$v_i$ is a virtual ground that is $v_i = 0$, thus the current flowing through the 10-kΩ resistor connected between $v_i$ and ground is zero. Therefore,
$v_o = v_i - R \times 0.5\text{ mA} = 0 - 10\text{ k}\Omega \times 0.5\text{ mA}$
$= -5$ V.

**Ex: 2.6**




$v_1$ is a virtual ground, thus $v_1 = 0$ V

$$i_1 = \frac{2\text{ V} - v_1}{R_1} = \frac{2 - 0}{1\text{ k}\Omega} = 2\text{ mA}$$

Assuming an ideal op amp, the current flowing into the negative input terminal of the op amp is zero. Therefore, $i_2 = i_1 \Rightarrow i_2 = 2$ mA

$$v_O = v_1 - i_2 R_2 = 0 - 2\text{ mA} \times 5\text{ k}\Omega = -10\text{ V}$$

$$i_L = \frac{v_o}{R_L} = \frac{-10\text{ V}}{1\text{ k}\Omega} = -10\text{ mA}$$

$$i_O = i_L - i_2 = -10\text{ mA} - 2\text{ mA} = -12\text{ mA}$$

Voltage gain $= \dfrac{v_O}{2\text{ V}} = \dfrac{-10\text{ V}}{1\text{ V}} = -5$ V/V or 14 dB

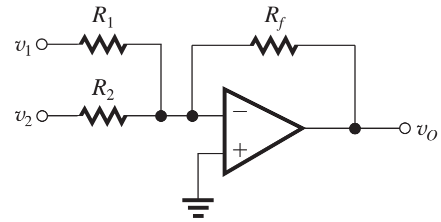
Current gain $= \dfrac{i_L}{i_1} = \dfrac{-10\text{ mA}}{2\text{ mA}} = -5$ A/A or 14 dB

Power gain

$$= \frac{P_L}{P_i} = \frac{-10(-10\text{ mA})}{2\text{ V} \times 2\text{ mA}} = 25\text{ W/W or 14 dB}$$

Note that power gain in dB is $10 \log_{10} \left| \dfrac{P_L}{P_i} \right|$.

**Ex: 2.7**




For the circuit shown above we have:

$$v_O = -\left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 \right)$$

Since it is required that $v_O = -(v_1 + 4v_2)$,

we want to have:

$$\frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 4$$

It is also desired that for a maximum output voltage of 4 V, the current in the feedback resistor not exceed 1 mA.

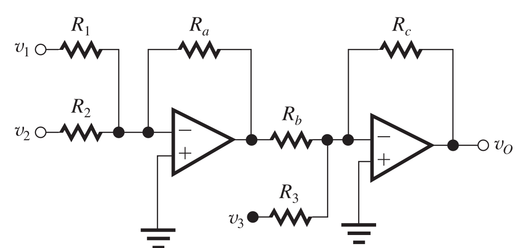
Therefore

$$\frac{4\text{ V}}{R_f} \leq 1\text{ mA} \Rightarrow R_f \geq \frac{4\text{ V}}{1\text{ mA}} \Rightarrow R_f \geq 4\text{ k}\Omega$$

Let us choose $R_f$ to be 4 kΩ, then

$R_1 = R_f = 4$ kΩ and $R_2 = \dfrac{R_f}{4} = 1$ kΩ

**Ex: 2.8**




$$v_O = \left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right)v_1 + \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right)v_2 - \left(\frac{R_c}{R_3}\right)v_3$$

We want to design the circuit such that

$$v_O = 2v_1 + v_2 - 4v_3$$

Thus we need to have

$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1, \text{ and } \frac{R_c}{R_3} = 4$$

From the above three equations, we have to find six unknown resistors; therefore, we can arbitrarily choose three of these resistors. Let us choose $R_a = R_b = R_c = 10$ kΩ.

Then we have

$R_3 = \frac{R_c}{4} = \frac{10}{4} = 2.5\ \text{k}\Omega$

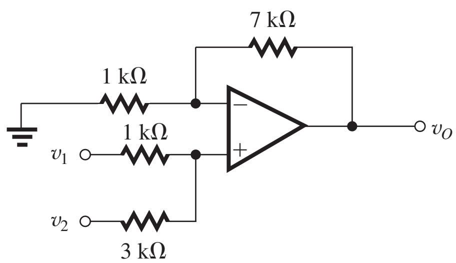
$\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) = 2, \Rightarrow \frac{10}{R_1} \times \frac{10}{10} = 2$

$\Rightarrow R_1 = 5\ \text{k}\Omega$

$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1$

$\Rightarrow R_2 = 10\ \text{k}\Omega$

**Ex: 2.9** Using the superposition principle to find the contribution of $v_1$ to the output voltage $v_O$, we set $v_2 = 0$




$v_+$ (the voltage at the positive input of the op amp

is: $v_+ = \frac{3}{1+3} v_1 = 0.75 v_1$

Thus $v_O = \left(1 + \frac{7\ \text{k}\Omega}{1\ \text{k}\Omega}\right) v_+ = 8 \times 0.75 v_1 = 6 v_1$

To find the contribution of $v_2$ to the output voltage $v_O$ we set $v_1 = 0$.

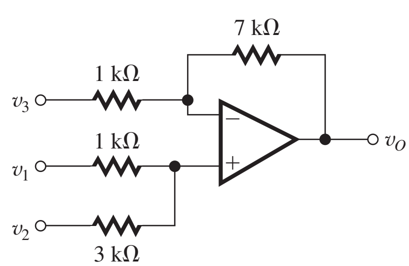
Then $v_+ = \frac{1}{1+3} v_2 = 0.25 v_2$

Hence
$v_O = \left(1 + \frac{7\ \text{k}\Omega}{1\ \text{k}\Omega}\right) v_+ = 8 \times 0.25 v_2 = 2 v_2$

Combining the contributions of $v_1$ and $v_2$

to $v_O$, we have $v_O = 6v_1 + 2v_2$

**Ex: 2.10**




Using the superposition principle to find the contribution of $v_1$ to $v_O$, we set $v_2 = v_3 = 0$. Then we have (refer to the solution of Exercise 2.9): $v_O = 6v_1$

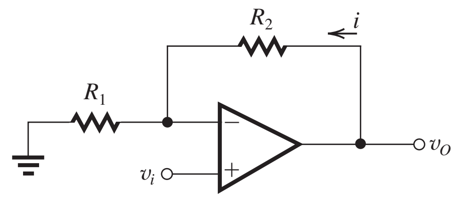
To find the contribution of $v_2$ to $v_O$, we set $v_1 = v_3 = 0$, then: $v_O = 2v_2$

To find the contribution of $v_3$ to $v_O$ we set $v_1 = v_2 = 0$, then

$v_o = -\frac{7\ \text{k}\Omega}{1\ \text{k}\Omega} v_3 = -7v_3$

Combining the contributions of $v_1, v_2,$ and $v_3$ to $v_O$ we have: $v_O = 6v_1 + 4v_2 - 7v_3$.

**Ex: 2.11**




$\frac{v_O}{v_i} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$

If $v_O = 10$ V, then it is desired that $i = 10\ \mu\text{A}$.

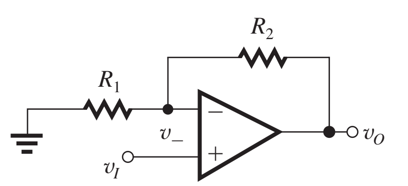
Thus,
$i = \frac{10\ \text{V}}{R_1 + R_2} = 10\ \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10\ \text{V}}{10\ \mu\text{A}}$

$R_1 + R_2 = 1\ \text{M}\Omega$ and

$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5\ \text{M}\Omega$

**Ex: 2.12**

(a)




$v_I - v_- = v_O/A \Rightarrow v_- = v_I - v_O/A$ (1)

But from the voltage divider across $v_O$,

$v_- = v_O \frac{R_1}{R_1 + R_2}$ (2)

Equating Eq. (1) and Eq. (2) gives

$v_O \frac{R_1}{R_1 + R_2} = v_I - \frac{v_O}{A}$

which can be manipulated to the form

$$\frac{v_O}{v_I} = \frac{1 + (R_2/R_1)}{1 + \dfrac{1 + (R_2/R_1)}{A}}$$

(b) For $R_1 = 1\ \text{k}\Omega$ and $R_2 = 9\ \text{k}\Omega$ the ideal value for the closed-loop gain is $1 + \frac{9}{1}$, that is, 10. The actual closed-loop gain is $G = \frac{10}{1 + 10/A}$.

If $A = 10^3$, then $G = 9.901$ and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For $v_I = 1$ V, $v_O = G \times v_I = 9.901$ V and

$$v_O = A(v_+ - v_-) \Rightarrow v_+ - v_- = \frac{v_O}{A} = \frac{9.901}{1000}$$

$\simeq 9.9$ mV

If $A = 10^4$, then $G = 9.99$ and $\epsilon = -0.1\%$.

For $v_I = 1$ V, $v_O = G \times v_I = 9.99$ V,

therefore,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.99}{10^4} = 0.999\ \text{mV} \simeq 1\ \text{mV}$$

If $A = 10^5$, then $G = 9.999$ and $\epsilon = -0.01\%$

For $v_I = 1$ V, $v_O = G \times v_I = 9.999$ thus,

$$v_+ - v_- = \frac{v_O}{A} = \frac{9.999}{10^5} = 0.09999\ \text{mV}$$

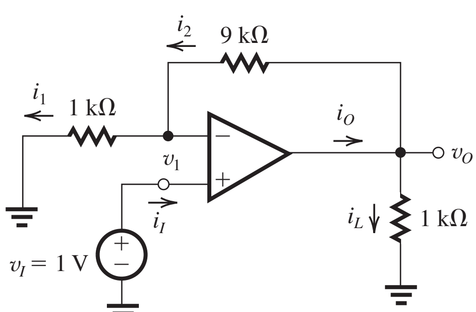
$\simeq 0.1$ mV

**Ex: 2.13**

$i_I = 0$ A, $v_1 = v_I = 1$ V

$$i_1 = \frac{v_1}{1\ \text{k}\Omega} = \frac{1\ \text{V}}{1\ \text{k}\Omega} = 1\ \text{mA}$$

$i_2 = i_1 = 1$ mA




$$v_O = v_1 + i_2 \times 9\ \text{k}\Omega = 1 + 1 \times 9 = 10\ \text{V}$$

$$i_L = \frac{v_O}{1\ \text{k}\Omega} = \frac{10\ \text{V}}{1\ \text{k}\Omega} = 10\ \text{mA}$$

$i_O = i_L + i_2 = 11$ mA

$$\frac{v_O}{v_i} = \frac{10\ \text{V}}{1\ \text{V}} = 10\ \text{V/V or 20 dB}$$

$$\frac{i_L}{i_I} = \frac{10\ \text{mA}}{0} = \infty$$

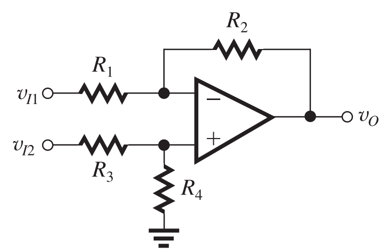
$$\frac{P_L}{P_I} = \frac{v_O \times i_L}{v_I \times i_I} = \frac{10 \times 10}{1 \times 0} = \infty$$

**Ex: 2.14**

(a) Load voltage $= \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 1\ \text{M}\Omega} \times 1\ \text{V} \simeq 1\ \text{mV}$

(b) Load voltage = 1 V

**Ex: 2.15**




(a) $R_1 = R_3 = 2\ \text{k}\Omega$, $R_2 = R_4 = 200\ \text{k}\Omega$

Since $R_4/R_3 = R_2/R_1$ we have:

$$A_d = \frac{v_O}{v_{I2} - v_{I1}} = \frac{R_2}{R_1} = \frac{200}{2} = 100\ \text{V/V}$$

(b) $R_{id} = 2R_1 = 2 \times 2\ \text{k}\Omega = 4\ \text{k}\Omega$

Since we are assuming the op amp is ideal,

$R_o = 0\ \Omega$

(c)

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The worst-case common-mode gain (i.e., the largest $A_{cm}$) occurs when the resistor tolerances are such that the quantity in parentheses is maximum. This in turn occurs when $R_2$ and $R_3$ are at their highest possible values (each one percent above nominal) and $R_1$ and $R_4$ are at their lowest possible values (each one percent below nominal), resulting in

$$A_{cm} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{1.01 \times 1.01}{0.99 \times 0.99}\right)$$

$$|A_{cm}| \simeq \frac{R_4}{R_3 + R_4} \times 0.04 \simeq \frac{200}{202} \times 0.04 \simeq 0.04\ \text{V/V}$$

The corresponding CMRR is

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.04} = 2500$$

or 68 dB.

**Ex: 2.16** We choose $R_3 = R_1$ and $R_4 = R_2$. Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of 20 kΩ, we require

$$A_d = \frac{R_2}{R_1} = 10 \text{ and}$$

$$R_{Id} = 2R_1 = 20\text{ k}\Omega \Rightarrow R_1 = 10\text{ k}\Omega \text{ and}$$

$$R_2 = A_d R_1 = 10 \times 10\text{ k}\Omega = 100\text{ k}\Omega$$

Therefore, $R_1 = R_3 = 10\text{ k}\Omega$ and

$R_2 = R_4 = 100\text{ k}\Omega$.

**Ex: 2.17** Given $v_{Icm} = +5$ V

$$v_{Id} = 10 \sin \omega t \text{ mV}$$

$$2R_1 = 1\text{ k}\Omega,\ R_2 = 0.5\text{ M}\Omega$$

$$R_3 = R_4 = 10\text{ k}\Omega$$

$$v_{I1} = v_{Icm} - \frac{1}{2}v_{Id} = 5 - \frac{1}{2} \times 0.01 \sin \omega t$$

$$= 5 - 0.005 \sin \omega t \text{ V}$$

$$v_{I2} = v_{Icm} + \frac{1}{2}v_{Id}$$

$$= 5 + 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_1) = v_{I1} = 5 - 0.005 \sin \omega t \text{ V}$$

$$v_-(\text{op amp } A_2) = v_{I2} = 5 + 0.005 \sin \omega t \text{ V}$$

$$v_{Id} = v_{I2} - v_{I1} = 0.01 \sin \omega t$$

$$v_{O1} = v_{I1} - R_2 \times \frac{v_{Id}}{2R_1}$$

$$= 5 - 0.005 \sin \omega t - 500\text{ k}\Omega \times \frac{0.01 \sin \omega t}{1\text{ k}\Omega}$$

$$= (5 - 5.005 \sin \omega t) \text{ V}$$

$$v_{O2} = v_{I2} + R_2 \times \frac{v_{Id}}{2R_1}$$

$$= (5 + 5.005 \sin \omega t) \text{ V}$$

$$v_+(\text{op amp } A_3) = v_{O2} \times \frac{R_4}{R_3 + R_4} = v_{O2}\frac{10}{10 + 10}$$

$$= \frac{1}{2}v_{O2} = \frac{1}{2}(5 + 5.005 \sin \omega t)$$

$$= (2.5 + 2.5025 \sin \omega t)\text{V}$$

$$v_-(\text{op amp } A_3) = v_+(\text{op amp } A_3)$$

$$= (2.5 + 2.5025 \sin \omega t) \text{ V}$$

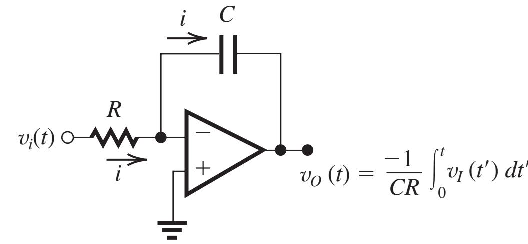
$$v_O = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$\frac{10\text{ k}\Omega}{10\text{ k}\Omega}\left(1 + \frac{0.5\text{ M}\Omega}{0.5\text{ k}\Omega}\right) \times 0.01 \sin \omega t$$

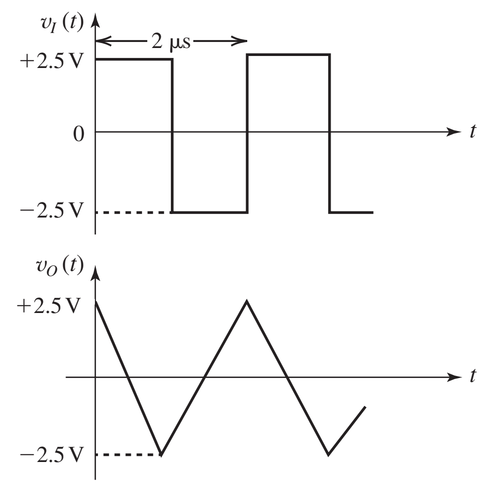
$$= 1(1 + 1000) \times 0.01 \sin \omega t$$

$$= 10.01 \sin \omega t \text{ V}$$

**Ex: 2.18**




The signal waveforms will be as shown.



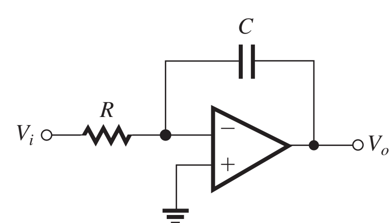

When $v_I = +2.5$ V, the current through the capacitor will be in the direction indicated, $i = 2.5 \text{ V}/R$, and the output voltage will decrease linearly from +2.5 V to −2.5 V. Thus in $(T/2)$ seconds, the capacitor voltage changes by 5 V. The charge equilibrium equation can be expressed as

$$i(T/2) = C \times 5 \text{ V}$$

$$\frac{2.5}{R}\,\frac{T}{2} = 5C \Rightarrow CR = \frac{2.5T}{10} = \frac{1}{4} \times 2 \times 10^{-6}$$

$$= 0.5\ \mu\text{s}$$

**Ex: 2.19**




The input resistance of this inverting integrator is $R$; therefore, $R = 10\text{ k}\Omega$.

Since the desired integration time constant is $10^{-3}$ s, we have: $CR = 10^{-3}$ s $\Rightarrow$

$$C = \frac{10^{-3}\text{ s}}{10\text{ k}\Omega} = 0.1\ \mu\text{F}$$

From Eq. (2.27) the transfer function of this integrator is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

For $\omega = 10$ rad/s, the integrator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 100\text{ V/V}$$

and phase $\phi = 90°$.

For $\omega = 1$ rad/s, the integrator transfer function has magnitude

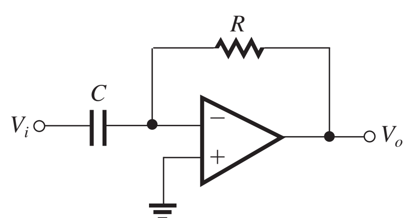
$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 \times 10^{-3}} = 1000\text{ V/V}$$

and phase $\phi = 90°$.

The frequency at which the integrator gain magnitude is unity is

$$\omega_{\text{int}} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000\text{ rad/s}$$

**Ex: 2.20**




$C = 0.01\ \mu$F is the input capacitance of this differentiator. We want $CR = 10^{-2}$ s (the time constant of the differentiator); thus,

$$R = \frac{10^{-2}}{0.01\ \mu\text{F}} = 1\text{ M}\Omega$$

From Eq. (2.33), the transfer function of the differentiator is

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Thus, for $\omega = 10$ rad/s the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10 \times 10^{-2} = 0.1\text{ V/V}$$

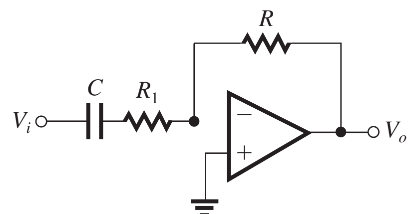
and phase $\phi = -90°$.

For $\omega = 10^3$ rad/s, the differentiator transfer function has magnitude

$$\left|\frac{V_o}{V_i}\right| = 10^3 \times 10^{-2} = 10\text{ V/V}$$

and phase $\phi = -90°$.

If we add a resistor in series with the capacitor to limit the high-frequency gain of the differentiator to 100, the circuit would be:




At high frequencies the capacitor $C$ acts like a short circuit. Therefore, the high-frequency gain of this circuit is: $\frac{R}{R_1}$. To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_1} = 100 \Rightarrow R_1 = \frac{R}{100} = \frac{1\text{ M}\Omega}{100} = 10\text{ k}\Omega$$

**Ex: 2.21**

Refer to the model in Fig. 2.29 and observe that

$$v_+ - v_- = V_{OS} + v_2 - v_1 = V_{OS} + v_{Id}$$

and since $v_O = v_3 = A(v_+ - v_-)$, then

$$v_O = A(v_{Id} + V_{OS}) \qquad (1)$$

where $A = 10^4$ V/V and $V_{OS} = 5$ mV. From Eq. (1) we see that $v_{id} = 0$ results in $v_O = 50$ V, which is impossible; thus the op amp saturates and $v_O = +10$ V. This situation pertains for $v_{Id} \geq -4$ mV. If $v_{Id}$ decreases below $-4$ mV, the op-amp output decreases correspondingly. For instance, $v_{Id} = -4.5$ mV results in $v_O = +5$ V; $v_{Id} = -5$ mV results in $v_O = 0$ V; $v_{Id} = -5.5$ mV results in $v_O = -5$ V; and $v_{Id} = -6$ mV results in $v_O = -10$ V, at which point the op amp saturates at the negative level of $-10$ V. Further decreases in $v_{Id}$ have no effect on the output voltage. The result is the transfer characteristic sketched in Fig. E2.21. Observe that the linear range of the characteristic is now centered around $v_{Id} = -5$ mV rather than the ideal situation of $v_{Id} = 0$; this shift is obviously a result of the input offset voltage $V_{OS}$.