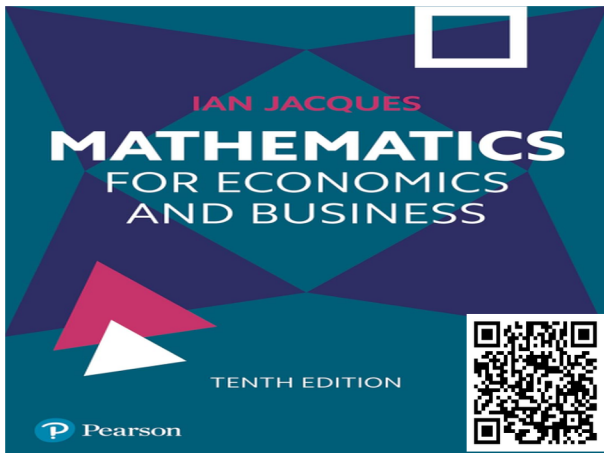


Mathematics for Economics and Business Ian Jacques 10th
Edition PDF

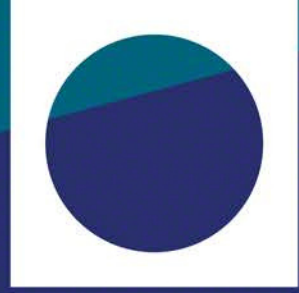
Visit the link below to download the full version of the ebook

[DOWNLOAD NOW](#)



Scan to Download
or Type the Link

ebook.ac/mathematics10e



IAN JACQUES

MATHEMATICS

FOR ECONOMICS AND BUSINESS

TENTH EDITION

MATHEMATICS

FOR ECONOMICS AND BUSINESS



Pearson

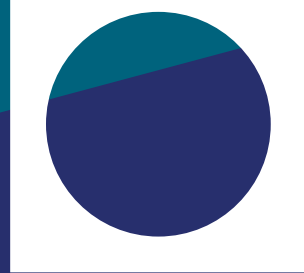
At Pearson, we have a simple mission: to help people make more of their lives through learning.

We combine innovative learning technology with trusted content and educational expertise to provide engaging and effective learning experiences that serve people wherever and whenever they are learning.

From classroom to boardroom, our curriculum materials, digital learning tools and testing programmes help to educate millions of people worldwide – more than any other private enterprise.

Every day our work helps learning flourish, and wherever learning flourishes, so do people.

To learn more, please visit us at www.pearson.com/uk



MATHEMATICS

FOR ECONOMICS AND BUSINESS

IAN JACQUES

TENTH EDITION



Pearson

Harlow, England • London • New York • Boston • San Francisco • Toronto • Sydney • Dubai • Singapore • Hong Kong
Tokyo • Seoul • Taipei • New Delhi • Cape Town • São Paulo • Mexico City • Madrid • Amsterdam • Munich • Paris • Milan

PEARSON EDUCATION LIMITED

KAO Two
KAO Park
Harlow CM17 9NA
United Kingdom
Tel: +44 (0)1279 623623
Web: www.pearson.com/uk

First published 1991 (print)
Second edition published 1994 (print)
Third edition published 1999 (print)
Fourth edition published 2003 (print)
Fifth edition published 2006 (print)
Sixth edition published 2009 (print)
Seventh edition published 2013 (print and electronic)
Eighth edition published 2015 (print and electronic)
Ninth edition published 2018 (print and electronic)
Tenth edition published 2023 (print and electronic)

- © Addison-Wesley Publishers Ltd 1991, 1994 (print)
- © Pearson Education Limited 1999, 2009 (print)
- © Pearson Education Limited 2013, 2015, 2018, 2023 (print and electronic)

The right of Ian Jacques to be identified as author of this work has been asserted by him in accordance with the Copyright, Designs and Patents Act 1988.

The print publication is protected by copyright. Prior to any prohibited reproduction, storage in a retrieval system, distribution or transmission in any form or by any means, electronic, mechanical, recording or otherwise, permission should be obtained from the publisher or, where applicable, a licence permitting restricted copying in the United Kingdom should be obtained from the Copyright Licensing Agency Ltd, Barnard's Inn, 86 Fetter Lane, London EC4A 1EN.

The ePublication is protected by copyright and must not be copied, reproduced, transferred, distributed, leased, licensed or publicly performed or used in any way except as specifically permitted in writing by the publishers, as allowed under the terms and conditions under which it was purchased, or as strictly permitted by applicable copyright law. Any unauthorised distribution or use of this text may be a direct infringement of the author's and the publisher's rights and those responsible may be liable in law accordingly.

All trademarks used herein are the property of their respective owners. The use of any trademark in this text does not vest in the author or publisher any trademark ownership rights in such trademarks, nor does the use of such trademarks imply any affiliation with or endorsement of this book by such owners.

Pearson Education is not responsible for the content of third-party internet sites.

ISBN: 978-1-292-72012-8 (print)
978-1-292-45182-4 (PDF)
978-1-292-72014-2 (ePub)

British Library Cataloguing-in-Publication Data

A catalogue record for the print edition is available from the British Library

Library of Congress Cataloging-in-Publication Data

10 9 8 7 6 5 4 3 2 1
27 26 25 24 23

Front cover image: Design Deluxe

Print edition typeset in 10/12.5pt Sabon MT Pro by Straive

Printed in Slovakia by Neografia

NOTE THAT ANY PAGE CROSS REFERENCES REFER TO THE PRINT EDITION

To my family: Past, Present and Future

CONTENTS

Preface	xiii
INTRODUCTION: Getting Started	1
Notes for students: how to use this text	1
Tour of textbook features	4
CHAPTER 1 Linear Equations	7
1.1 Introduction to algebra	8
1.1.1 Negative numbers	9
1.1.2 Expressions	11
1.1.3 Brackets	14
Key Terms	19
Exercise 1.1	20
Exercise 1.1*	22
1.2 Further algebra	24
1.2.1 Fractions	24
1.2.2 Equations	31
1.2.3 Inequalities	35
Key Terms	38
Exercise 1.2	38
Exercise 1.2*	40
1.3 Graphs of linear equations	42
Key Terms	52
Exercise 1.3	52
Exercise 1.3*	54
1.4 Algebraic solution of simultaneous linear equations	55
Key Term	65
Exercise 1.4	65
Exercise 1.4*	66
1.5 Supply and demand analysis	67
Key Terms	80
Exercise 1.5	80
Exercise 1.5*	82
1.6 Transposition of formulae	84
Key Terms	91
Exercise 1.6	91
Exercise 1.6*	92
1.7 National income determination	93
Key Terms	105
Exercise 1.7	105
Exercise 1.7*	106
Case study 1	109
Formal mathematics	110
Multiple-choice questions	113
Examination questions	117

CHAPTER 2	Non-linear Equations	123
2.1	Quadratic functions	124
	Key Terms	138
	Exercise 2.1	139
	Exercise 2.1*	140
2.2	Revenue, cost and profit	142
	Key Terms	150
	Exercise 2.2	150
	Exercise 2.2*	151
2.3	Indices and logarithms	153
	2.3.1 Index notation	153
	2.3.2 Rules of indices	157
	2.3.3 Logarithms	163
	2.3.4 Summary	169
	Key Terms	170
	Exercise 2.3	170
	Exercise 2.3*	172
2.4	The exponential and natural logarithm functions	175
	Key Terms	185
	Exercise 2.4	185
	Exercise 2.4*	186
	Case study 2	189
	Formal mathematics	191
	Multiple-choice questions	194
	Examination questions	198
CHAPTER 3	Mathematics of Finance	203
3.1	Percentages	204
	3.1.1 Index numbers	210
	3.1.2 Inflation	214
	Key Terms	216
	Exercise 3.1	216
	Exercise 3.1*	219
3.2	Compound interest	222
	Key Terms	232
	Exercise 3.2	232
	Exercise 3.2*	234
3.3	Geometric series	236
	Key Terms	244
	Exercise 3.3	244
	Exercise 3.3*	245
3.4	Investment appraisal	247
	Key Terms	259
	Exercise 3.4	259
	Exercise 3.4*	261
	Case study 3	263
	Formal mathematics	264
	Multiple-choice questions	266
	Examination questions	270

CHAPTER 4	Differentiation	275
4.1	The derivative of a function	276
	Key Terms	285
	Exercise 4.1	285
	Exercise 4.1*	286
4.2	Rules of differentiation	287
	Rule 1 The constant rule	287
	Rule 2 The sum rule	288
	Rule 3 The difference rule	289
	Key Terms	294
	Exercise 4.2	294
	Exercise 4.2*	296
4.3	Marginal functions	298
	4.3.1 Revenue and cost	298
	4.3.2 Production	305
	4.3.3 Consumption and savings	307
	Key Terms	309
	Exercise 4.3	309
	Exercise 4.3*	310
4.4	Further rules of differentiation	312
	Rule 4 The chain rule	313
	Rule 5 The product rule	315
	Rule 6 The quotient rule	318
	Exercise 4.4	320
	Exercise 4.4*	321
4.5	Elasticity	322
	Key Terms	334
	Exercise 4.5	334
	Exercise 4.5*	335
4.6	Optimisation of economic functions	337
	Key Terms	353
	Exercise 4.6	353
	Exercise 4.6*	355
4.7	Further optimisation of economic functions	356
	Key Term	367
	Exercise 4.7*	367
4.8	The derivative of the exponential and natural logarithm functions	369
	Exercise 4.8	378
	Exercise 4.8*	379
	Case study 4	381
	Formal mathematics	384
	Multiple-choice questions	387
	Examination questions	393
CHAPTER 5	Partial Differentiation	399
5.1	Functions of several variables	400
	Key Terms	410
	Exercise 5.1	411
	Exercise 5.1*	412
5.2	Partial elasticity and marginal functions	414
	5.2.1 Elasticity of demand	414

5.2.2	Utility	417
5.2.3	Production	423
	Key Terms	425
	Exercise 5.2	426
	Exercise 5.2*	428
5.3	Comparative statics	430
	Key Terms	439
	Exercise 5.3*	439
5.4	Unconstrained optimisation	443
	Key Terms	454
	Exercise 5.4	454
	Exercise 5.4*	455
5.5	Constrained optimisation	457
	Key Terms	466
	Exercise 5.5	467
	Exercise 5.5*	468
5.6	Lagrange multipliers	470
	Key Terms	479
	Exercise 5.6	479
	Exercise 5.6*	480
	Case study 5	482
	Formal mathematics	483
	Multiple-choice questions	485
	Examination questions	488
CHAPTER 6	Integration	493
6.1	Indefinite integration	494
	Key Terms	505
	Exercise 6.1	506
	Exercise 6.1*	507
6.2	Definite integration	509
6.2.1	Consumer's surplus	513
6.2.2	Producer's surplus	514
6.2.3	Investment flow	516
6.2.4	Discounting	518
6.2.5	Income inequality	518
	Key Terms	520
	Exercise 6.2	520
	Exercise 6.2*	522
	Case study 6	524
	Formal mathematics	525
	Multiple-choice questions	527
	Examination questions	530
CHAPTER 7	Matrices	535
7.1	Basic matrix operations	536
7.1.1	Transposition	538
7.1.2	Addition and subtraction	539
7.1.3	Scalar multiplication	542
7.1.4	Matrix multiplication	543
7.1.5	Summary	551

Key Terms	551
Exercise 7.1	552
Exercise 7.1*	554
7.2 Matrix inversion	556
Key Terms	571
Exercise 7.2	571
Exercise 7.2*	572
7.3 Cramer's rule	575
Key Term	583
Exercise 7.3	583
Exercise 7.3*	584
Case study 7	587
Formal mathematics	589
Multiple-choice questions	590
Examination questions	594
CHAPTER 8 Linear Programming	599
8.1 Graphical solution of linear programming problems	600
Key Terms	614
Exercise 8.1	615
Exercise 8.1*	616
8.2 Applications of linear programming	618
Key Terms	626
Exercise 8.2	626
Exercise 8.2*	628
Case study 8	631
Formal mathematics	632
Multiple-choice questions	633
Examination questions	638
CHAPTER 9 Dynamics	643
9.1 Difference equations	644
9.1.1 National income determination	650
9.1.2 Supply and demand analysis	652
Key Terms	655
Exercise 9.1	655
Exercise 9.1*	656
9.2 Differential equations	659
9.2.1 National income determination	665
9.2.2 Supply and demand analysis	667
Key Terms	669
Exercise 9.2	670
Exercise 9.2*	671
Case study 9	674
Formal mathematics	675
Multiple-choice questions	676
Examination questions	679

Answers to Problems

	682
Chapter 1	682
Chapter 2	692
Chapter 3	701
Chapter 4	705
Chapter 5	716
Chapter 6	723
Chapter 7	727
Chapter 8	733
Chapter 9	737
Glossary	741
Index	748

Pearson's Commitment to Diversity, Equity and Inclusion

Pearson is dedicated to creating bias-free content that reflects the diversity, depth and breadth of all learners' lived experiences. We embrace the many dimensions of diversity including, but not limited to, race, ethnicity, gender, sex, sexual orientation, socioeconomic status, ability, age and religious or political beliefs.

Education is a powerful force for equity and change in our world. It has the potential to deliver opportunities that improve lives and enable economic mobility. As we work with authors to create content for every product and service, we acknowledge our responsibility to demonstrate inclusivity and incorporate diverse scholarship so that everyone can achieve their potential through learning. As the world's leading learning company, we have a duty to help drive change and live up to our purpose to help more people create a better life for themselves and to create a better world.

Our ambition is to purposefully contribute to a world where:

- Everyone has an equitable and lifelong opportunity to succeed through learning.
- Our educational products and services are inclusive and represent the rich diversity of learners.
- Our educational content accurately reflects the histories and lived experiences of the learners we serve.
- Our educational content prompts deeper discussions with students and motivates them to expand their own learning and worldview.

We are also committed to providing products that are fully accessible to all learners. As per Pearson's guidelines for accessible educational Web media, we test and retest the capabilities of our products against the highest standards for every release, following the WCAG guidelines in developing new products for copyright year 2022 and beyond. You can learn more about Pearson's commitment to accessibility at:

<https://www.pearson.com/us/accessibility.html>

While we work hard to present unbiased, fully accessible content, we want to hear from you about any concerns or needs regarding this Pearson product so that we can investigate and address them.

- Please contact us with concerns about any potential bias at:
<https://www.pearson.com/report-bias.html>
- For accessibility-related issues, such as using assistive technology with Pearson products, alternative text requests, or accessibility documentation, email the Pearson Disability Support team at:
disability.support@pearson.com

PREFACE

This textbook is intended primarily for students on economics, business studies and management courses. It assumes very little prerequisite knowledge, so it can be read by students who have not undertaken a mathematics course for some time. The style is informal, and the text contains a large number of worked examples. Students are encouraged to tackle problems for themselves as they read through each section. Solutions are provided so that all answers can be checked. Consequently, it should be possible to work through this text on a self-study basis. The material is wide ranging and varies from elementary topics such as percentages and linear equations to more sophisticated topics such as constrained optimisation of multivariate functions. The text should therefore be suitable for use on both low- and high-level quantitative methods courses.

This text was first published in 1991. The prime motivation for writing it then was to try to produce a book that students could actually read and understand for themselves. This remains the guiding principle when writing this tenth edition. Extra features and resources requested by students over the years are now incorporated into the printed book. These include end-of-chapter multiple-choice tests and longer examination questions. All of these are designed to help students pass the course.

Online resources include access to the eBook and a bank of new problems in MyLab Math. The latter provides students with an interactive learning environment where they can concentrate on topics that they find difficult and measure their own progress. There is a ‘help me solve it’ mode which provides step-by-step guidance to solve a problem with the system acting like a personal tutor. Lecturer’s resources include fully worked solutions to all of the standard problems in the textbook, PowerPoint slides, and material on more advanced topics.

Many of the examples and questions in this edition have been updated to make them more contemporary. New material on the second-order conditions for Lagrange multipliers and a new section on Lorenz curves and Gini coefficients are provided. Case studies are now included to give an indication how the mathematics covered in each chapter can actually be used by businesses and individuals in practice, which we hope students will find relevant and interesting.

Ian Jacques



INTRODUCTION

Getting Started

NOTES FOR STUDENTS: HOW TO USE THIS TEXT

I am always amazed by the mix of students on first-year economics courses. Some have not acquired any mathematical knowledge beyond elementary algebra (and even that can be of a rather dubious nature), some have never studied economics before in their lives, while others have passed preliminary courses in both. Whatever category you are in, I hope that you will find this text of value. The chapters covering algebraic manipulation, simple calculus, finance, matrices and linear programming should also benefit students on business studies and management courses.

The first few chapters are aimed at complete beginners and students who have not taken mathematics courses for some time. I would like to think that these students once enjoyed mathematics and had every intention of continuing their studies in this area, but somehow never found the time to fit it into an already overcrowded academic timetable. However, I suspect that the reality is rather different. Possibly they hated the subject, could not understand it and dropped it at the earliest opportunity. If you find yourself in this position, you are probably horrified to discover that you must embark on a quantitative methods course with an examination looming on the horizon. However, there is no need to worry. My experience is that every student is capable of passing a mathematics examination. All that is required is a commitment to study and a willingness to suspend any prejudices about the subject gained at school. The fact that you have bothered to buy this text at all suggests that you are prepared to do both.

To help you get the most out of this text, let me compare the working practices of economics and engineering students. The former rarely read individual books in any great depth. They tend to skim through a selection of books in the university library and perform a large number of Internet searches, picking out relevant information. Indeed, the ability to read selectively and to compare various sources of information is an important skill that all arts and social science students must acquire. Engineering students, on the other hand, are more likely to read just a few books in any one year. They read each of these from cover to cover and attempt virtually every problem en route. Even though you are most definitely not an engineer, it is the engineering approach that you need to adopt while studying mathematics. There are several reasons for this. First, a mathematics text can never be described, even by its most ardent admirers, as a good bedtime read. It can take an hour or two of concentrated effort to understand just a few pages of a mathematics text. You are therefore recommended to work through this text systematically in short bursts rather than to attempt to read whole chapters. Each section is designed to take between one and two hours to complete, and this is quite sufficient for a single session. Secondly, mathematics is a hierarchical subject in which one topic follows on from the next. A construction firm building an office block is hardly likely to erect the fiftieth storey without making sure that the intermediate floors and foundations are securely in place. Likewise, you cannot 'dip' into the middle of a mathematics text and expect to follow it unless you have satisfied the prerequisites for that topic. Finally, you actually need to do mathematics yourself before you can understand it. No matter how wonderful your lecturer is, and no matter how many problems are discussed in class, it is only

by solving problems yourself that you are ever going to become confident in using and applying mathematical techniques. For this reason, several problems are interspersed within the text, and you are encouraged to tackle these as you go along. You will require writing paper, graph paper, pens and a calculator for this. There is no need to buy an expensive calculator unless you are feeling particularly wealthy at the moment. A bottom-of-the-range **scientific** calculator should be good enough. Answers to every question are printed at the back of this text so that you can check your own answers quickly as you go along. However, please avoid the temptation to look at them until you have made an honest attempt at each one. Remember that in the future you may well have to sit down in an uncomfortable chair, in front of a blank sheet of paper, and be expected to produce solutions to examination questions of a similar type.

At the end of each section there are two parallel exercises. The non-starred exercises are intended for students who are meeting these topics for the first time and the questions are designed to consolidate basic principles. The starred exercises are more challenging but still cover the full range so that students with greater experience will be able to concentrate their efforts on these questions without having to pick-and-mix from both exercises. The chapter dependence is shown in Figure I.1. If you have studied some advanced mathematics before, you will discover that parts of Chapters 1, 2 and 4 are familiar. However, you may find that the sections on economics applications contain new material. You are best advised to test yourself by attempting a selection of problems from the starred exercise in each section to see if you need to read through it as part of a refresher course. Economics students in a desperate hurry to experience the delights of calculus can miss out Chapter 3 without any loss of continuity and move straight on to Chapter 4. The mathematics of finance is probably more relevant to business and accountancy students, although you can always read it later if it is part of your economics syllabus.

At the end of every chapter, you will find a multiple-choice test and some examination questions. These cover the work of the whole chapter. We recommend that you try the multiple-choice questions when you have completed the relevant chapter. As usual, answers

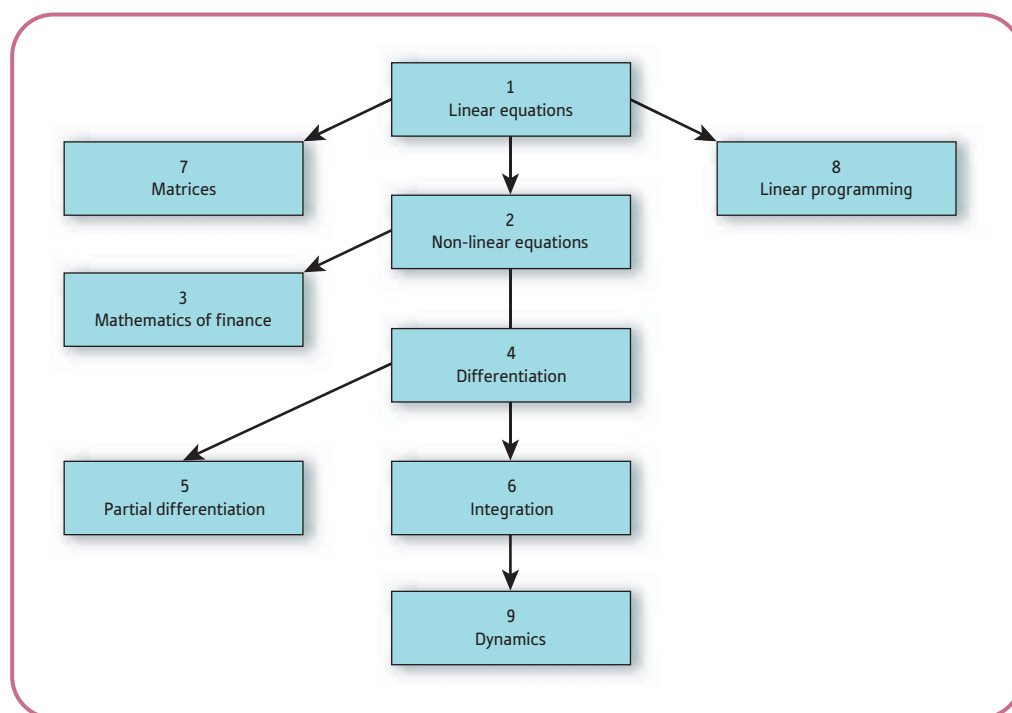


Figure I.1

are provided at the back of the book so that you can check to see how well you have done. If you do get any of the questions wrong, it would be worth redoing them, perhaps writing down full working so that you can spot your mistake more easily. The final section contains several examination-style problems which are more challenging. They tend to be longer than the questions encountered so far in the exercises and require more confidence and experience. You might like to leave these until the end of the course and use them in your build-up to the final exams.

I hope that this text helps you to succeed in your mathematics course. You never know, you might even enjoy it. Remember to wear your engineer's hat while reading the text. I have done my best to make the material as accessible as possible. The rest is up to you!

TOUR OF TEXTBOOK FEATURES

Objectives

At the end of this section you should be able to:

- Add, subtract, multiply and divide negative numbers.
- Understand what is meant by an algebraic expression.
- Evaluate algebraic expressions numerically.
- Simplify algebraic expressions by collecting like terms.
- Multiply out brackets.

Example

- (a) Find the value of $2x - 3y$ when $x = 9$ and $y = 4$.
 (b) Find the value of $2Q^2 + 4Q + 150$ when $Q = 5$.
 (c) Find the value of $5a - 2b + c$ when $a = 4$, $b = 3$ and $c = 2$.
 (d) Find the value of $(12 - t) - (t - 1)$ when $t = 8$.

Practice Problem

1. (1) Without using a calculator, evaluate

(a) $5 \times (-6)$ (b) $(-1) \times (-2)$

(d) $(-5) \div (-1)$ (e) $2 \times (-1) \times (-3)$

(2) Confirm your answer to part (1) using a calculator.

Advice

In this example the solutions are written out in painstaking detail to show how the distributive law is applied. The solutions to all three parts show only one or two steps of working. You are, of course, at liberty to write your own solutions, but please do not be tempted to overdo this. You may find this later date and may find it difficult if you have tried to be too clever.

Key Terms

- Autonomous consumption** The level of consumption when income is zero.
Autonomous savings The withdrawals from savings when income is zero.
Consumption function The relationship between national income and consumption.
Disposable income Household income after the deduction of taxes.

Objectives

Each section begins with a set of bullet points outlining the mathematical skills to be learnt. Use it as a checklist of things that you need to know.

Examples

Examples take centre stage in this book and there are lots of them. There is little point in understanding the theory unless you can apply it in practice. All examples are fully worked with every step carefully explained.

Practice problems

As their name suggests, these are provided so that you can practise skills that you have just learnt. They are interspersed within the text itself and provide an opportunity to take a break from just passive reading. They help you to consolidate understanding as you go along.

Advice

Advice boxes provide some helpful hints and guidance on particular issues and topics.

Key terms

Each section concludes with a list of key terms defining all the new mathematical terminology covered in that section. These are collected in a glossary at the end of the book which acts as a kind of dictionary of words and phrases in case you need a quick reminder.

Exercise 1.1

1. Without using a calculator, evaluate

(a) $10 \times (-2)$ (b) $(-1) \times (-3)$

(e) $24 \div (-2)$ (f) $(-10) \times (-5)$

Exercise 1.2*

1. Simplify each of the following algebraic fractions

(a) $\frac{2x-6}{4}$ (b) $\frac{9x}{6x^2-3x}$ (c) $\frac{4x}{x^2-6x+6}$

(d) $\frac{2x-6}{(x+3)(2x-5)}$

Case study 1

A start-up business, AccFin, aims to supply financial software to help small firms pick up contracts from small firms employing just a handful of people. AccFin provides pricing structure for companies which employ several thousand. To cater for this pricing structure:

Under Tariff 1: AccFin provides a site licence which covers

Exercises

Every section concludes with a standard exercise. Please remember that you cannot just learn maths by reading books or watching others solve problems. Try as many questions as you can in each exercise, checking your answers with those provided at the back of the book. This is your opportunity to take control of your own learning and boost your confidence.

Exercises*

If you find that you are familiar with some of the topics, then you should have a go at the starred exercises instead. They cover the same material as the standard exercise, so you may not need to bother with both. The questions are more challenging, so they should help to maintain your interest if you have covered the topic in a previous life.

Case studies

The case studies apply the mathematical techniques to a particular business or personal situation. They show how some of the ideas covered in each chapter can be put into practice.

Formal mathematics

The approach adopted in this text is very informal. Emphasis is placed on both the mathematics and economic applications as well as on how to help you to understand and enjoy the subject. However, much of the mathematics is built upon more theoretical foundations and notation, which are described in these end-of-chapter sections.

Formal mathematics

As far as possible this textbook has been written in a relaxed style with the sole purpose of trying to make the mathematics easy to understand. However, much of the mathematics is built upon more theoretical foundations and notation, which are described in these end-of-chapter sections.

Multiple-choice questions**Question 1**

A graph of the demand function, $3P + 4Q = 9$ is plotted with Q on the horizontal axis and P on the vertical axis. Find the slope of the line and the intercept on the P axis.

Multiple-choice questions

Once you get to the end of each chapter, you might like to know that you have got the hang of all topics covered. End-of-chapter multiple-choice questions provide a quick way of doing this (without the tedium of having to write out your solutions neatly).

Examination questions**Question 1**

A function of two variables is given by $f(x, y) = 2x + 3x^2y - y^3$. Find expressions for the first-order and second-order partial derivatives of f with respect to x and y .

Examination questions

These are longer questions designed to prepare you for your end-of-year exam. You might like to save them until the end of the course, after you have had a chance to revise properly.



CHAPTER 1

Linear Equations

The main aim of this chapter is to introduce the mathematics of linear equations. This is an obvious first choice in an introductory text, since it is an easy topic which has many applications. There are seven sections, which are intended to be read in the order that they appear.

Sections 1.1, 1.2, 1.3, 1.4 and 1.6 are devoted to mathematical methods. They serve to revise the rules of arithmetic and algebra, which you probably met at school but may have forgotten. In particular, the properties of negative numbers and fractions are considered. A reminder is given on how to multiply out brackets and how to manipulate mathematical expressions. You are also shown how to solve simultaneous linear equations. Systems of two equations in two unknowns can be solved using graphs, which are described in Section 1.3. However, the preferred method uses elimination, which is considered in Section 1.4. This algebraic approach has the advantage that it always gives an exact solution and it extends readily to larger systems of equations.

The remaining two sections are reserved for applications in microeconomics and macroeconomics. You may be pleasantly surprised by how much economic theory you can analyse using just the basic mathematical tools considered here. Section 1.5 introduces the fundamental concept of an economic function and describes how to calculate equilibrium prices and quantities in supply and demand theory. Section 1.7 deals with national income determination in simple macroeconomic models.

The first six sections underpin the rest of the text and are essential reading. The final section is not quite as important and can be omitted at this stage.

SECTION 1.1

Introduction to algebra

Objectives

At the end of this section you should be able to:

- Add, subtract, multiply and divide negative numbers.
- Understand what is meant by an algebraic expression.
- Evaluate algebraic expressions numerically.
- Simplify algebraic expressions by collecting like terms.
- Multiply out brackets.
- Factorise algebraic expressions.

ALGEBRA IS BORING

There is no getting away from the fact that algebra *is* boring. Doubtless there are a few enthusiasts who get a kick out of algebraic manipulation, but economics and business students are rarely to be found in this category. Indeed, the mere mention of the word ‘algebra’ is enough to strike fear into the heart of many a first-year student. Unfortunately, you cannot get very far with mathematics unless you have completely mastered this topic. An apposite analogy is the game of chess. Before you can begin to play a game of chess, it is necessary to go through the tedium of learning the moves of individual pieces. In the same way it is essential that you learn the rules of algebra before you can enjoy the ‘game’ of mathematics. Of course, just because you know the rules does not mean that you are going to excel at the game, and no one is expecting you to become a grandmaster of mathematics. However, you should at least be able to follow the mathematics presented in economics books and journals as well as to solve simple problems for yourself.

Advice

If you have studied mathematics recently, then you will find the material in the first few sections of the text fairly straightforward. You may prefer just to try the questions in the starred exercise at the end of each section to get yourself back up to speed. However, if it has been some time since you have studied this subject, our advice is very different. Please work through the material thoroughly even if it is vaguely familiar. Make sure that you do the problems as they arise, checking your answers with those provided at the back of this text. The material has been broken down into three subsections:

- negative numbers;
- expressions;
- brackets.

You might like to work through these subsections on separate occasions to enable the ideas to sink in. To rush this topic now is likely to give you only a half-baked understanding, which will result in hours of frustration when you study the later chapters of this text.

1.1.1 Negative numbers

In mathematics, numbers are classified into one of three types: positive, negative or zero. At school you were probably introduced to the idea of a negative number via the temperature on a thermometer scale measured in degrees centigrade. A number such as -5 would then be interpreted as a temperature of 5 degrees below freezing. In personal finance a negative bank balance would indicate that an account is 'in the red' or 'in debit'. Similarly, a firm's profit of $-500\,000$ signifies a loss of half a million.

The rules for the multiplication of negative numbers are

$$\boxed{\text{negative}} \times \boxed{\text{negative}} = \boxed{\text{positive}}$$

$$\boxed{\text{negative}} \times \boxed{\text{positive}} = \boxed{\text{negative}}$$

It does not matter in which order two numbers are multiplied, so

$$\boxed{\text{positive}} \times \boxed{\text{negative}} = \boxed{\text{negative}}$$

These rules produce, respectively,

$$(-2) \times (-3) = 6$$

$$(-4) \times 5 = -20$$

$$7 \times (-5) = -35$$

Also, because division is the same sort of operation as multiplication (it just undoes the result of multiplication and takes you back to where you started), exactly the same rules apply when one number is divided by another. For example,

$$(-15) \div (-3) = 5$$

$$(-16) \div 2 = -8$$

$$2 \div (-4) = -1/2$$

In general, to multiply or divide lots of numbers it is probably simplest to ignore the signs to begin with and just to work the answer out. The final result is negative if the total number of minus signs is odd and positive if the total number is even.

Example

Evaluate

$$(a) (-2) \times (-4) \times (-1) \times 2 \times (-1) \times (-3) \quad (b) \frac{5 \times (-4) \times (-1) \times (-3)}{(-6) \times 2}$$

Solution

(a) Ignoring the signs gives

$$2 \times 4 \times 1 \times 2 \times 1 \times 3 = 48$$

There are an odd number of minus signs (in fact, five), so the answer is -48 .

(b) Ignoring the signs gives

$$\frac{5 \times 4 \times 1 \times 3}{6 \times 2} = \frac{60}{12} = 5$$

There are an even number of minus signs (in fact, four), so the answer is 5.

Advice

Attempt the following problem yourself both with and without a calculator. On most machines a negative number such as -6 is entered by pressing the button labelled $(-)$ followed by 6.

Practice Problem

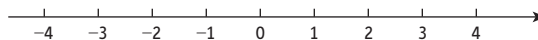
1. (1) Without using a calculator, evaluate

(a) $5 \times (-6)$ (b) $(-1) \times (-2)$ (c) $(-50) \div 10$

(d) $(-5) \div (-1)$ (e) $2 \times (-1) \times (-3) \times 6$ (f) $\frac{2 \times (-1) \times (-3) \times 6}{(-2) \times 3 \times 6}$

(2) Confirm your answer to part (1) using a calculator.

To add or subtract negative numbers it helps to think in terms of a number line:



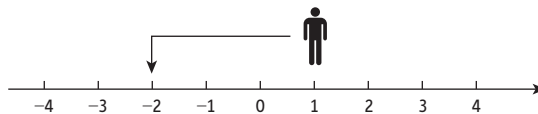
If b is a positive number, then

$$a - b$$

can be thought of as an instruction to start at a and to move b units to the left. For example,

$$1 - 3 = -2$$

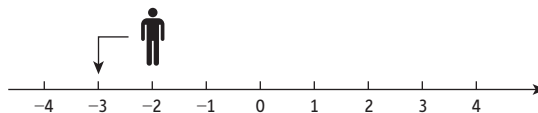
because if you start at 1 and move 3 units to the left, you end up at -2 :



Similarly,

$$-2 - 1 = -3$$

because 1 unit to the left of -2 is -3 .



On the other hand,

$$a - (-b)$$

is taken to be $a + b$. This follows from the rule for multiplying two negative numbers, since

$$-(-b) = (-1) \times (-b) = b$$

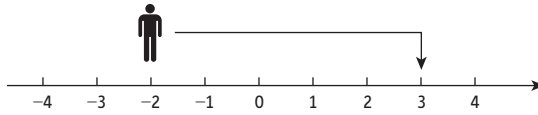
Consequently, to evaluate

$$a - (-b)$$

you start at a and move b units to the right (that is, in the positive direction). For example,

$$-2 - (-5) = -2 + 5 = 3$$

because if you start at -2 and move 5 units to the right, you end up at 3.



Practice Problem

2. (1) Without using a calculator, evaluate

(a) $1 - 2$ (b) $-3 - 4$ (c) $1 - (-4)$

(d) $-1 - (-1)$ (e) $-72 - 19$ (f) $-53 - (-48)$

(2) Confirm your answer to part (1) using a calculator.

1.1.2 Expressions

In algebra, letters are used to represent numbers. In pure mathematics the most common letters used are x and y . However, in applications it is helpful to choose letters that are more meaningful, so we might use Q for quantity and I for investment. An algebraic expression is then simply a combination of these letters, brackets and other mathematical symbols such as $+$ or $-$. For example, the expression

$$P \left(1 + \frac{r}{100} \right)^n$$

can be used to work out how money in a savings account grows over a period of time. The letters P , r and n represent the original sum invested (called the principal – hence the use of the letter P), the rate of interest and the number of years, respectively. To work it all out, you not only need to replace these letters by actual numbers, but you also need to understand the various conventions that go with algebraic expressions such as this.

In algebra, when we multiply two numbers represented by letters, we usually suppress the multiplication sign between them. The product of a and b would simply be written as ab without bothering to put the multiplication sign between the symbols. Likewise, when a number represented by the letter Y is doubled, we write $2Y$. In this case we not only suppress the multiplication sign but adopt the convention of writing the number in front of the letter. Here are some further examples:

$P \times Q$ is written as PQ

$d \times 8$ is written as $8d$

$n \times 6 \times t$ is written as $6nt$

$z \times z$ is written as z^2 (using the index 2 to indicate squaring a number)

$1 \times t$ is written as t (since multiplying by 1 does not change a number)

In order to evaluate these expressions it is necessary to be given the numerical value of each letter. Once this has been done, you can work out the final value by performing the operations in the following order:

- | | |
|-----------------------------------|------|
| Brackets first | (B) |
| Indices second | (I) |
| Division and Multiplication third | (DM) |
| Addition and Subtraction fourth | (AS) |

This is sometimes remembered using the acronym BIDMAS, and it is essential to use this ordering for working out all mathematical calculations. For example, suppose you wish to evaluate each of the following expressions when $n = 3$:

$$2n^2 \text{ and } (2n)^2$$

Substituting $n = 3$ into the first expression gives

$$\begin{aligned} 2n^2 &= 2 \times 3^2 && \text{(the multiplication sign is revealed when we switch from algebra to numbers)} \\ &= 2 \times 9 && \text{(according to BIDMAS, indices are worked out before multiplication)} \\ &= 18 \end{aligned}$$

whereas in the second expression we get

$$\begin{aligned} (2n)^2 &= (2 \times 3)^2 && \text{(again, the multiplication sign is revealed)} \\ &= 6^2 && \text{(according to BIDMAS, we evaluate the inside of the brackets first)} \\ &= 36 \end{aligned}$$

The two answers are not the same, so the order indicated by BIDMAS really does matter. Looking at the previous list, notice that there is a tie between multiplication and division for third place, and another tie between addition and subtraction for fourth place. These pairs of operations have equal priority, and under these circumstances you work from left to right when evaluating expressions. For example, substituting $x = 5$ and $y = 4$ in the expression, $x - y + 2$, gives

$$\begin{aligned} x - y + 2 &= 5 - 4 + 2 \\ &= 1 + 2 && \text{(reading from left to right, subtraction comes first)} \\ &= 3 \end{aligned}$$

Example

- (a) Find the value of $2x - 3y$ when $x = 9$ and $y = 4$.
- (b) Find the value of $2Q^2 + 4Q + 150$ when $Q = 10$.
- (c) Find the value of $5a - 2b + c$ when $a = 4$, $b = 6$ and $c = 1$.
- (d) Find the value of $(12 - t) - (t - 1)$ when $t = 4$.

Solution

$$\begin{aligned} \text{(a) } 2x - 3y &= 2 \times 9 - 3 \times 4 && \text{(substituting numbers)} \\ &= 18 - 12 && \text{(multiplication has priority over subtraction)} \\ &= 6 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 2Q^2 + 4Q + 150 &= 2 \times 10^2 + 4 \times 10 + 150 && \text{(substituting numbers)} \\
 &= 2 \times 100 + 4 \times 10 + 150 && \text{(indices have priority over multiplication and addition)} \\
 &= 200 + 40 + 150 && \text{(multiplication has priority over addition)} \\
 &= 390
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 5a - 2b + c &= 5 \times 4 - 2 \times 6 + 1 && \text{(substituting numbers)} \\
 &= 20 - 12 + 1 && \text{(multiplication has priority over addition and subtraction)} \\
 &= 8 + 1 && \text{(addition and subtraction have equal priority, so work from left to right)} \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad (12 - t) - (t - 1) &= (12 - 4) - (4 - 1) && \text{(substituting numbers)} \\
 &= 8 - 3 && \text{(brackets first)} \\
 &= 5
 \end{aligned}$$

Practice Problem

3. Evaluate each of the following by replacing the letters by the given numbers:

(a) $2Q + 5$ when $Q = 7$.

(b) $5x^2y$ when $x = 10$ and $y = 3$.

(c) $4d - 3f + 2g$ when $d = 7$, $f = 2$ and $g = 5$.

(d) $a(b + 2c)$ when $a = 5$, $b = 1$ and $c = 3$.

Like terms are multiples of the same letter (or letters). For example, $2P$, $-34P$ and $0.3P$ are all multiples of P and so are like terms. In the same way, xy , $4xy$ and $69xy$ are all multiples of xy and so are like terms. If an algebraic expression contains like terms which are added or subtracted together, then it can be simplified to produce an equivalent shorter expression.

Example

Simplify each of the following expressions (where possible):

(a) $2a + 5a - 3a$

(b) $4P - 2Q$

(c) $3w + 9w^2 + 2w$

(d) $3xy + 2y^2 + 9x + 4xy - 8x$

Solution

(a) All three are like terms since they are all multiples of a , so the expression can be simplified:

$$2a + 5a - 3a = 4a$$



(b) The terms $4P$ and $2Q$ are unlike because one is a multiple of P and the other is a multiple of Q , so the expression cannot be simplified.

(c) The first and last are like terms since they are both multiples of w , so we can collect these together and write

$$3w + 9w^2 + 2w = 5w + 9w^2$$

This cannot be simplified any further because $5w$ and $9w^2$ are unlike terms.

(d) The terms $3xy$ and $4xy$ are like terms, and $9x$ and $8x$ are also like terms. These pairs can therefore be collected together to give

$$3xy + 2y^2 + 9x + 4xy - 8x = 7xy + 2y^2 + x$$

Notice that we write just x instead of $1x$ and also that no further simplification is possible since the final answer involves three unlike terms.

Practice Problem

4. Simplify each of the following expressions, where possible:

(a) $2x + 6y - x + 3y$

(b) $5x + 2y - 5x + 4z$

(c) $4Y^2 + 3Y - 43$

(d) $8r^2 + 4s - 6rs - 3s - 3s^2 + 7rs$

(e) $2e^2 + 5f - 2e^2 - 9f$

(f) $3w + 6W$

(g) $ab - ba$

1.1.3 Brackets

It is useful to be able to take an expression containing brackets and rewrite it as an equivalent expression without brackets, and vice versa. The process of removing brackets is called ‘expanding brackets’ or ‘multiplying out brackets’. This is based on the **distributive law**, which states that for any three numbers a , b and c

$$a(b + c) = ab + ac$$

It is easy to verify this law in simple cases. For example, if $a = 2$, $b = 3$ and $c = 4$, then the left-hand side is

$$2(3 + 4) = 2 \times 7 = 14$$

However,

$$ab = 2 \times 3 = 6 \quad \text{and} \quad ac = 2 \times 4 = 8$$

and so the right-hand side is $6 + 8$, which is also 14.

This law can be used when there are any number of terms inside the brackets. We have

$$a(b + c + d) = ab + ac + ad$$

$$a(b + c + d + e) = ab + ac + ad + ae$$

and so on.

It does not matter in which order two numbers are multiplied, so we also have

$$(b + c)a = ba + ca$$

$$(b + c + d)a = ba + ca + da$$

$$(b + c + d + e)a = ba + ca + da + ea$$

Example

Multiply out the brackets in

(a) $x(x - 2)$

(b) $2(x + y - z) + 3(z + y)$

(c) $x + 3y - (2y + x)$

Solution

(a) The use of the distributive law to multiply out $x(x - 2)$ is straightforward. The x outside the bracket multiplies the x inside to give x^2 . The x outside the bracket also multiplies the -2 inside to give $-2x$. Hence

$$x(x - 2) = x^2 - 2x$$

(b) To expand

$$2(x + y - z) + 3(z + y)$$

we need to apply the distributive law twice. We have

$$2(x + y - z) = 2x + 2y - 2z$$

$$3(z + y) = 3z + 3y$$

Adding together gives

$$\begin{aligned} 2(x + y - z) + 3(z + y) &= 2x + 2y - 2z + 3z + 3y \\ &= 2x + 5y + z \quad (\text{collecting like terms}) \end{aligned}$$

(c) It may not be immediately apparent how to expand

$$x + 3y - (2y + x)$$

However, note that

$$-(2y + x)$$

is the same as

$$(-1)(2y + x)$$

which expands to give

$$(-1)(2y) + (-1)x = -2y - x$$

Hence

$$x + 3y - (2y + x) = x + 3y - 2y - x = y$$

after collecting like terms.

Advice

In this example the solutions are written out in painstaking detail. This is done to show you precisely how the distributive law is applied. The solutions to all three parts could have been written down in only one or two steps of working. You are, of course, at liberty to compress the working in your own solutions, but please do not be tempted to overdo this. You might want to check your answers at a later date and may find it difficult if you have tried to be too clever.

Practice Problem

5. Multiply out the brackets, simplifying your answer as far as possible.

(a) $(5 - 2z)z$ (b) $6(x - y) + 3(y - 2x)$ (c) $x - y + z - (x^2 + x - y)$

Mathematical formulae provide a precise way of representing calculations that need to be worked out in many business models. However, it is important to realise that these formulae may be valid only for a restricted range of values. Most large companies have a policy to reimburse employees for use of their cars for travel: for the first 50 miles they may be able to claim 90 cents a mile, but this could fall to 60 cents a mile thereafter. If the distance, x miles, is no more than 50 miles, then travel expenses, E (in dollars), could be worked out using the formula $E = 0.9x$. If x exceeds 50 miles, the employee can claim \$0.90 a mile for the first 50 miles but only \$0.60 a mile for the last $(x - 50)$ miles. The total amount is then

$$\begin{aligned} E &= 0.9 \times 50 + 0.6(x - 50) \\ &= 45 + 0.6x - 30 \\ &= 15 + 0.6x \end{aligned}$$

Travel expenses can therefore be worked out using two separate formulae:

- $E = 0.9x$ when x is no more than 50 miles
- $E = 15 + 0.6x$ when x exceeds 50 miles.

Before we leave this topic, a word of warning is in order. Be careful when removing brackets from very simple expressions such as those considered in part (c) in the previous worked example and practice problem. A common mistake is to write

$$(a + b) - (c + d) = a + b - c + d \quad \text{This is NOT true}$$

The distributive law tells us that the -1 multiplying the second bracket applies to the d as well as the c , so the correct answer has to be

$$(a + b) - (c + d) = a + b - c - d$$

In algebra, it is sometimes useful to reverse the procedure and put the brackets back in. This is called **factorisation**. Consider the expression $12a + 8b$. There are many numbers which divide into both 8 and 12. However, we always choose the biggest number, which is 4 in this case, so we attempt to take the factor of 4 outside the brackets:

$$12a + 8b = 4(? + ?)$$

where each ? indicates a mystery term inside the brackets. We would like 4 multiplied by the first term in the brackets to be $12a$, so we are missing $3a$. Likewise, if we are to generate an $8b$, the second term in the brackets will have to be $2b$.

Hence

$$12a + 8b = 4(3a + 2b)$$

As a check, notice that when you expand the brackets on the right-hand side, you really do get the expression on the left-hand side.

Example

Factorise

(a) $6L - 3L^2$

(b) $5a - 10b + 20c$

Solution

(a) Both terms have a common factor of 3. Also, because $L^2 = L \times L$, both $6L$ and $-3L^2$ have a factor of L . Hence we can take out a common factor of $3L$ altogether.

$$6L - 3L^2 = 3L(2) - 3L(L) = 3L(2 - L)$$

(b) All three terms have a common factor of 5, so we write

$$5a - 10b + 20c = 5(a) - 5(2b) + 5(4c) = 5(a - 2b + 4c)$$

Practice Problem

6. Factorise

(a) $7d + 21$

(b) $16w - 20q$

(c) $6x - 3y + 9z$

(d) $5Q - 10Q^2$

We conclude our discussion of brackets by describing how to multiply two brackets together. In the expression $(a + b)(c + d)$ the two terms a and b must each multiply the single bracket $(c + d)$, so

$$(a + b)(c + d) = a(c + d) + b(c + d)$$

The first term $a(c + d)$ can itself be expanded as $ac + ad$. Likewise, $b(c + d) = bc + bd$. Hence

$$(a + b)(c + d) = ac + ad + bc + bd$$

This procedure then extends to brackets with more than two terms:

$$(a + b)(c + d + e) = a(c + d + e) + b(c + d + e) = ac + ad + ae + bc + bd + be$$

Example

Multiply out the brackets

(a) $(x + 1)(x + 2)$ (b) $(x + 5)(x - 5)$ (c) $(2x - y)(x + y - 6)$

simplifying your answer as far as possible.

Solution

$$\begin{aligned} \text{(a)} \quad (x + 1)(x + 2) &= x(x + 2) + (1)(x + 2) \\ &= x^2 + 2x + x + 2 \\ &= x^2 + 3x + 2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (x + 5)(x - 5) &= x(x - 5) + 5(x - 5) \\ &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25 \end{aligned}$$

the xs cancel

$$\begin{aligned} \text{(c)} \quad (2x - y)(x + y - 6) &= 2x(x + y - 6) - y(x + y - 6) \\ &= 2x^2 + 2xy - 12x - yx - y^2 + 6y \\ &= 2x^2 + xy - 12x - y^2 + 6y \end{aligned}$$

Practice Problem

7. Multiply out the brackets.

(a) $(x + 3)(x - 2)$

(b) $(x + y)(x - y)$

(c) $(x + y)(x + y)$

(d) $(5x + 2y)(x - y + 1)$

Looking back at part (b) of the previous worked example, notice that

$$(x + 5)(x - 5) = x^2 - 25 = x^2 - 5^2$$

Quite generally

$$\begin{aligned} (a + b)(a - b) &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - b^2 \end{aligned}$$

The result

$$a^2 - b^2 = (a + b)(a - b)$$

is called the **difference of two squares** formula. It provides a quick way of factorising certain expressions.

Example

Factorise the following expressions:

(a) $x^2 - 16$ (b) $9x^2 - 100$

Solution

(a) Noting that

$$x^2 - 16 = x^2 - 4^2$$

we can use the difference of two squares formula to deduce that

$$x^2 - 16 = (x + 4)(x - 4)$$

(b) Noting that

$$9x^2 - 100 = (3x)^2 - (10)^2$$

$$(3x)^2 = 3x \times 3x = 9x^2$$

we can use the difference of two squares formula to deduce that

$$9x^2 - 100 = (3x + 10)(3x - 10)$$

Practice Problem

8. Factorise the following expressions:

(a) $x^2 - 64$ (b) $4x^2 - 81$

Advice

This completes your first piece of mathematics. We hope that you have not found it quite as bad as you first thought. There now follow a few extra problems to give you more practice. Not only will they help to strengthen your mathematical skills, but also they should improve your overall confidence. Two alternative exercises are available. Exercise 1.1 is suitable for students whose mathematics may be rusty and who need to consolidate their understanding. Exercise 1.1* contains more challenging problems and so is more suitable for those students who have found this section very easy.

If you would like some extra help at any stage of your course, you might like to make use of MyLab Math. This online resource is directly linked to each section of this textbook. It includes a 'Help Me Solve This' option which provides step-by-step guidance on how to tackle each question.

Key Terms

Difference of two squares The algebraic result which states that $a^2 - b^2 = (a + b)(a - b)$.

Distributive law The law of arithmetic which states that $a(b + c) = ab + ac$ for any numbers, a, b, c .

Factorisation The process of writing an expression as a product of simpler expressions using brackets.

Like terms Multiples of the same combination of algebraic symbols.

Exercise 1.1

1. Without using a calculator, evaluate

(a) $10 \times (-2)$ (b) $(-1) \times (-3)$ (c) $(-8) \div 2$ (d) $(-5) \div (-5)$
 (e) $24 \div (-2)$ (f) $(-10) \times (-5)$ (g) $\frac{20}{-4}$ (h) $\frac{-27}{-9}$
 (i) $(-6) \times 5 \times (-1)$ (j) $\frac{2 \times (-6) \times 3}{(-9)}$

2. Without using a calculator, evaluate

(a) $5 - 6$ (b) $-1 - 2$ (c) $6 - 17$ (d) $-7 + 23$
 (e) $-7 - (-6)$ (f) $-4 - 9$ (g) $7 - (-4)$ (h) $-9 - (-9)$
 (i) $12 - 43$ (j) $2 + 6 - 10$

3. Without using a calculator, evaluate

(a) $5 \times 2 - 13$ (b) $\frac{-30 - 6}{-18}$ (c) $\frac{(-3) \times (-6) \times (-1)}{2 - 3}$ (d) $5 \times (1 - 4)$
 (e) $1 - 6 \times 7$ (f) $-5 + 6 \div 3$ (g) $2 \times (-3)^2$ (h) $-10 + 2^2$
 (i) $(-2)^2 - 5 \times 6 + 1$ (j) $\frac{(-4)^2 \times (-3) \times (-1)}{(-2)^3}$

4. The *monthly* running costs of energy for my studio flat vary throughout the year:

Month	Jan–Feb	Mar–Apr	May–Sept	Oct–Nov	Dec
Cost (\$)	255	210	90	210	255

I pay by a monthly direct debit of \$170 and the statement of my account on 1 January is listed as ‘-\$110’, where the ‘-’ sign indicates that I am in debit.

Work out what my statement will show by the end of

- (a) September (b) December

5. The average monthly temperature in °C for Barrow Point, Alaska, is as follows:

J	F	M	A	M	J	J	A	S	O	N	D
-26	-26	-25	-17	-6	2	5	4	0	-9	-17	-22

Work out the temperature change from

- (a) January to June (b) April to October (c) July to December

Models of climate change predict that these monthly averages will all rise by 3.5°C by the end of this century. What do these models predict the average temperature will be in March 2100?

6. Simplify each of the following algebraic expressions:

(a) $2 \times P \times Q$ (b) $I \times 8$ (c) $3 \times x \times y$
 (d) $4 \times q \times w \times z$ (e) $b \times b$ (f) $k \times 3 \times k$

7. Simplify the following algebraic expressions by collecting like terms:

(a) $6w - 3w + 12w + 4w$

(b) $6x + 5y - 2x - 12y$

(c) $3a - 2b + 6a - c + 4b - c$

(d) $2x^2 + 4x - x^2 - 2x$

(e) $2cd + 4c - 5dc$

(f) $5st + s^2 - 3ts + t^2 + 9$

8. Without using a calculator, find the value of the following:

(a) $2x - y$ when $x = 7$ and $y = 4$.

(b) $x^2 - 5x + 12$ when $x = 6$.

(c) $2m^3$ when $m = 10$.

(d) $5fg^2 + 2g$ when $f = 2$ and $g = 3$.

(e) $2v + 4w - (4v - 7w)$ when $v = 20$ and $w = 10$.

9. If $x = 2$ and $y = -3$, evaluate

(a) $2x + y$

(b) $x - y$

(c) $3x + 4y$

(d) xy

(e) $5xy$

(f) $4x - 6xy$

10. (a) Without using a calculator, work out the value of $(-4)^2$.

(b) Press the following key sequence on your calculator:

$$\boxed{(-)} \quad \boxed{4} \quad \boxed{x^2}$$

Explain carefully why this does not give the same result as part (a) and give an alternative key sequence that *does* give the correct answer.

11. Without using a calculator, work out

(a) $(5 - 2)^2$

(b) $5^2 - 2^2$

Is it true in general that $(a - b)^2 = a^2 - b^2$?

12. Use your calculator to work out the following. Round your answer, if necessary, to two decimal places.

(a) $5.31 \times 8.47 - 1.01^2$

(b) $(8.34 + 2.27)/9.41$

(c) $9.53 - 3.21 + 4.02$

(d) $2.41 \times 0.09 - 1.67 \times 0.03$

(e) $45.76 - (2.55 + 15.83)$

(f) $(3.45 - 5.38)^2$

(g) $4.56(9.02 + 4.73)$

(h) $6.85/(2.59 + 0.28)$

13. Multiply out the brackets:

(a) $7(x - y)$

(b) $3(5x - 2y)$

(c) $4(x + 3)$

(d) $7(3x - 1)$

(e) $3(x + y + z)$

(f) $x(3x - 4)$

(g) $y + 2z - 2(x + 3y - z)$

14. Factorise

(a) $25c + 30$

(b) $9x - 18$

(c) $x^2 + 2x$

(d) $16x - 12y$

(e) $4x^2 - 6xy$

(f) $10d - 15e + 50$

15. Multiply out the brackets:

(a) $(x + 2)(x + 5)$

(b) $(a + 4)(a - 1)$

(c) $(d + 3)(d - 8)$

(d) $(2s + 3)(3s + 7)$

(e) $(2y + 3)(y + 1)$

(f) $(5t + 2)(2t - 7)$

(g) $(3n + 2)(3n - 2)$

(h) $(a - b)(a - b)$



16. Simplify the following expressions by collecting together like terms:

(a) $2x + 3y + 4x - y$ (b) $2x^2 - 5x + 9x^2 + 2x - 3$

(c) $5xy + 2x + 9yx$ (d) $7xyz + 3yx - 2zyx + yzx - xy$

(e) $2(5a + b) - 4b$ (f) $5(x - 4y) + 6(2x + 7y)$

(g) $5 - 3(p - 2)$ (h) $x(x - y + 7) + xy + 3x$

17. Use the formula for the difference of two squares to factorise

(a) $x^2 - 4$ (b) $Q^2 - 49$ (c) $x^2 - y^2$ (d) $9x^2 - 100y^2$

18. Simplify the following algebraic expressions:

(a) $3x - 4x^2 - 2 + 5x + 8x^2$ (b) $x(3x + 2) - 3x(x + 5)$

19. A law firm seeks to recruit top-quality experienced lawyers. The total package offered is the sum of three separate components: a basic salary which is 1.2 times the candidate's current salary together with an additional \$3000 for each year worked as a qualified lawyer and an extra \$1000 for every year that they are over the age of 21.

Work out a formula that could be used to calculate the total salary, S , offered to someone who is A years of age, has E years of relevant experience and currently earns $\$N$. Hence work out the salary offered to someone who is 30 years old with five years' experience and who currently earns \$150 000.

20. Write down a formula for each situation:

(a) A plumber has a fixed call-out charge of \$100 and has an hourly rate of \$80. Work out the total charge, C , for a job that takes L hours in which the cost of materials and parts is $\$K$.

(b) An airport currency exchange booth charges a fixed fee of \$10 on all transactions and offers an exchange rate of 1 dollar to 0.8 euros. Work out the total charge, C (in \$) for buying x euros.

(c) A firm provides 5 hours of in-house training for each of its semi-skilled workers and 10 hours of training for each of its skilled workers. Work out the total number of hours, H , if the firm employs a semi-skilled and b skilled workers.

(d) A car hire company charges $\$C$ a day together with an additional $\$c$ per mile. Work out the total charge, $\$X$, for hiring a car for d days and travelling m miles during that time.

Exercise 1.1*

1. Without using a calculator, evaluate

(a) $(12 - 8) - (6 - 5)$ (b) $12 - (8 - 6) - 5$ (c) $12 - 8 - 6 - 5$

2. Put a pair of brackets in the left-hand side of each of the following to give correct statements:

(a) $2 - 7 - 9 + 3 = -17$ (b) $8 - 2 + 3 - 4 = -1$ (c) $7 - 2 - 6 + 10 = 1$

3. Without using a calculator, work out the value of each of the following expressions in the case when $a = 3$, $b = -4$ and $c = -2$:

(a) $a(b - c)$ (b) $3c(a + b)$ (c) $a^2 + 2b + 3c$ (d) $2abc^2$
 (e) $\frac{c + b}{2a}$ (f) $\sqrt{2(b^2 - c)}$ (g) $\frac{b}{2c} - \frac{a}{3b}$ (h) $5a - b^3 - 4c^2$

4. Without using a calculator, evaluate each of the following expressions in the case when $x = -1$, $y = -2$ and $z = 3$:

(a) $x^3 + y^2 + z$ (b) $\sqrt{\left(\frac{x^2 + y^2 + z}{x^2 + 2xy - z}\right)}$ (c) $\frac{xyz(x + z)(z - y)}{(x + y)(x - z)}$

5. Multiply out the brackets and simplify

$$(x - y)(x + y) - (x + 2)(x - y + 3)$$

6. Simplify

(a) $x - y - (y - x)$ (b) $x - ((y - x) - y)$ (c) $x + y - (x - y) - (x - (y - x))$

7. Multiply out the brackets:

(a) $(x + 4)(x - 6)$ (b) $(2x - 5)(3x - 7)$ (c) $2x(3x + y - 2)$
 (d) $(3 + g)(4 - 2g + h)$ (e) $(2x + y)(1 - x - y)$ (f) $(a + b + c)(a - b - c)$

8. Factorise

(a) $9x - 12y$ (b) $x^2 - 6x$ (c) $10xy + 15x^2$
 (d) $3xy^2 - 6x^2y + 12xy$ (e) $x^3 - 2x^2$ (f) $60x^4y^6 - 15x^2y^4 + 20xy^3$

9. Use the formula for the difference of two squares to factorise

(a) $p^2 - 25$ (b) $9c^2 - 64$ (c) $32v^2 - 50d^2$ (d) $16x^4 - y^4$

10. Evaluate the following without using a calculator:

(a) $50\,563^2 - 49\,437^2$ (b) $90^2 - 89.99^2$
 (c) $759^2 - 541^2$ (d) $123\,456\,789^2 - 123\,456\,788^2$

11. A specialist paint manufacturer receives \$20 for each pot sold. The initial set-up cost for the production run is \$1500 and the cost of making each pot of paint is \$5.

- (a) Write down a formula for the total profit, π , if the firm manufactures x pots of paint and sells y pots.
 (b) Use your formula to calculate the profit when $x = 1000$ and $y = 800$.
 (c) State any restrictions on the variables in the mathematical formula in part (a).
 (d) Simplify the formula in the case when the firm sells all that it manufactures.

12. Factorise

(a) $2KL^2 + 4KL$ (b) $L^2 - 0.04K^2$ (c) $K^2 + 2LK + L^2$

Independent variable A variable whose value determines that of the dependent variable; in $y = f(x)$, the independent variable is x .

Index Alternative word for exponent or power.

Index number The scale factor of a variable measured from the base year multiplied by 100.

Indifference curve A curve indicating all combinations of two goods which give the same level of utility.

Indifference map A diagram showing the graphs of a set of indifference curves. The further the curve is from the origin, the greater the level of utility.

Inelastic demand Where the percentage change in demand is less than the corresponding change in price: $|E| < 1$.

Inferior good A good whose demand decreases as income increases.

Inflation The percentage increase in the level of prices over a 12-month period.

Initial condition The value of Y_0 (or $y(0)$) which needs to be specified to obtain a unique solution of a difference (or differential) equation.

Integer programming A linear programming problem in which the search for solution is restricted to points in the feasible region with whole-number coordinates.

Integral The number $\int_a^b f(x)dx$ (definite integral) or the function $\int f(x)dx$ (indefinite integral).

Integration The generic name for the evaluation of definite or indefinite integrals.

Intercept The points where a graph crosses one of the co-ordinate axes.

Internal rate of return (IRR) The interest rate for which the net present value is zero.

Interval The set of all real numbers between (and possibly including) two given numbers.

Inverse (operation) The operation that reverses the effect of a given operation and takes you back to the original. For example, the inverse of halving is doubling.

Inverse function A function, written f^{-1} , which reverses the effect of a given function, f , so that $x = f^{-1}(y)$ when $y = f(x)$.

Inverse matrix A matrix A^{-1} with the property that $A^{-1}A = I = AA^{-1}$.

Investment The creation of output not for immediate consumption.

Investment multiplier The number by which you multiply the change in investment to deduce the corresponding change in, say, national income: $\partial Y/\partial I^*$.

IS schedule The equation relating national income and interest rate based on the assumption of equilibrium in the goods market.

Isocost curve A line showing all combinations of two factors which can be bought for a fixed cost.

Isoquant A curve indicating all combinations of two factors which give the same level of output.

L-shaped curve A term used by economists to describe the graph of a function, such as $f(x) = a + \frac{b}{x}$, which bends roughly like the letter L.

Labour All forms of human input to the production process.

Labour productivity Average output per worker: Q/L

Lagrange multiplier The number λ which is used in the Lagrangian function. In economics this gives the approximate change in the value of the objective function when the value of the constraint is increased by 1 unit.

Lagrangian function The function $f(x, y) + \lambda[M - \varphi(x, y)]$, where $f(x, y)$ is the objective function and $\varphi(x, y) = M$ is the constraint. The stationary point of this function is the solution of the associated constrained optimisation problem.

Laspeyres index An index number for groups of data which are weighted by the quantities used in the base year.

Law of diminishing marginal productivity (law of diminishing returns) Once the size of the workforce exceeds a particular value, the increase in output due to a 1-unit increase in labour will decline: $d^2Q/dL^2 < 0$ for sufficiently large L .

Law of diminishing marginal utility The law which states that the increase in utility due to the consumption of an additional good will eventually decline: $\partial^2 U/\partial x_i^2 < 0$ for sufficiently large x_i .

Like terms Multiples of the same combination of algebraic symbols.

Limited growth Used to describe an economic variable which increases over time but which tends to a fixed quantity.

Limits of integration The numbers a and b which appear in the definite integral, $\int_a^b f(x)dx$.

Linear equation An equation of the form $y = dx + f$.

LM schedule The equation relating national income and interest rate based on the assumption of equilibrium in the money market.

Logarithm The power to which a base must be raised to yield a particular number.

Lorenz curve A graph of $y = L(x)$ where $100y\%$ is the total income that is earned by the bottom $100x\%$ of the population.

INDEX

Note: Page numbers in bold refer to Glossary entries.

- absolute value 110, 112, **741**
- addition
 - fractions 27–9
 - matrices 539–41
 - negative numbers 10
- adjoint matrices 567, 568
- adjugate matrices 567, 568
- adjustment coefficients, differential equations 665, 669, **741**
- algebra 8–41
 - brackets 14–19
 - equations 31–7
 - fractions 24–31, **741**
 - inequalities 35–7
 - matrices 548, 556
 - transposition of formulae 84–92
- algebraic equations 31–7, 38
 - coefficients 44, 52, **741**
 - mathematical operations applied to 31–4
 - simultaneous linear equations 52, **746**
 - solving 47–8, 55–66
 - sketching lines from 44–52
- algebraic expressions 11–14
- algebraic fractions 24–31, **741**
 - addition of 27–9
 - differentiation of 317–19
 - division of 26–7
 - multiplication of 26–7
 - subtraction of 27–9
- annual compounding of interest 223–7
- annual equivalent rate (AER) 229
- annual percentage rate (APR) 229, 232, **741**
- annual rate of inflation 214–16
- annuities 252–3, 259, 518, **741**
- anti-derivatives 495, 505, **741**
 - see also* integrals
- APR *see* annual percentage rate
- arbitrary constants
 - in differential equations 659, 669, **741**
 - see also* constant of integration
- arc elasticity 324–5, 329, 334, **741**
- areas of graphs, finding by integration 509–15
 - unbounded regions 525–6
- arithmetic progression 237, 244, **741**
- associative law 548
 - in matrix algebra 548
- autonomous consumption 93, 105, **741**
 - autonomous consumption multiplier 431, 439, 562, **741**
 - autonomous export multiplier 435
 - autonomous savings 95, 105, **741**
 - autonomous taxation multiplier 434
 - average cost (AC) 144–6, 150, 349–50, 364, **741**
 - graphs 145, 146, 147
 - optimisation of 349–50, 364
 - average product of labour 345, 346, 353, **741**
 - optimisation of 345–6, 363–4
 - average revenue (AR) 302, 309, **741**
 - axes of graph 42, 52
- balanced budget multiplier 434, 439, **741**
- ‘balancing the equation’ approach 31–2
- base 153, 164, 167, 175
 - see also* logarithms; power(s)
- BIDMAS convention 12
 - applications 12, 131
- bonds, government 257–9
- brackets 14–19
- break-even points 147, 149
- budgets
 - balanced budget multiplier 434, 439, **741**
 - constraints 459–62
- calculus 275
 - differentiation 275–397
 - integration 493–532
 - partial differentiation 399–491
 - see also main entries:* differentiation; integration; partial differentiation
- capital 161, 170, 305, **741**
 - formation of 516–17
 - marginal product of 423, 425, **745**
- cartels 302
- CF *see* complementary functions
- chain rule of differentiation 312–13, 504–5
 - applications 314, 315, 316, 317, 372, 373, 377, 431, 437
- charts *see* flow charts; reverse flow charts
- chords of curves 299–300, 309, 384, 385, **741**
 - slope approaching that of tangent 384
- closed interval 110, 112, **741**
- Cobb–Douglas production functions 163, 170, 424, **741**
 - constrained optimisation of 475–7
- coefficient matrices 559
- coefficients in algebraic equations 44, 52, **741**
- cofactors (of matrix elements) 563–7, 571, **741**
- column vectors in matrices 539, 551, **741**
 - multiplication by row vectors 543–6
- columns in matrices 536, 538
- commodity prices 192
- commodity substitution, marginal rate of 420–2, 426, **745**
- common denominators 27, 28, 29
- common factors (in fractions) 26
- commutative law 548
 - in matrix algebra 549
- comparative statics 430–42
 - meaning of term 431, 439, **741**
- competition, perfect 69, 302–3, 309, 448, 515, **746**
- complementary functions (CF)
 - difference equations 645–6, 648, 649, 651, 653, 655, **741**
 - differential equations 661–2, 663, 664, 666, 668, 669, **741**
- complementary good(s) 72, 77, 80, **741**
- compound interest 222–35, **741**
 - annual compounding of 223–7
 - annual percentage rate and 229, 232
 - continuous compounding of 227–9, 232, 247, 518, **741**
 - discrete compounding of 223–7, 247, 248
 - exponential functions and 228
 - future value and 223, 225–6, 227
 - geometric series and 236
 - simple interest compared with 222
 - various compounding periods 226–7
- concave graphs 293, 294, **741**
- constant of integration 496, 505, 511, **741**
- constant returns to scale, production functions with 162, 163, 170, **741**
- constant rule of differentiation 287–8, 291, 498

- constrained optimisation 457–69, 606–14
 Lagrange multipliers 470–82
 linear programming 606–14
 method of substitution 460–2,
 466, 745
 objective functions 621–5
 constraints 457
 non-negativity *see* non-negativity
 constraints
 consumer's surplus 513–14, 520, 741
 consumption 93, 430
 autonomous 431, 439, 562, 741
 differentiation of 307, 308
 dynamics 650, 651
 marginal propensity to consume 93,
 95, 105, 307–8, 309, 745
see also equilibrium consumption;
 marginal propensity to consume
 consumption function 93, 94, 95, 97,
 105, 502–3, 741
 continuous compounding of interest
 227–9, 232, 247, 518, 742
 discount formula for 518
 continuous function 191, 192, 193, 741
 continuous revenue streams 518
 contour maps 419
 converges uniformly, meaning of term
 648–9, 650, 651, 652, 655
 convex graphs 293, 294, 742
 coordinates 42, 52, 742
 cost constraints 457, 458, 459, 462–4
 cost(s)
 average 144–6, 150, 349–50, 364, 741
 differentiation of 303–4, 309
 fixed 144, 150, 743
 holding 364–6
 marginal 303–4, 309, 745
 integration of 501–2, 503
 optimisation of economic
 functions 348, 356, 357–8
 optimisation of 349–50, 364
 linear programming used 621–5
 ordering 364–6
 total 142, 144, 145, 146, 150, 359,
 501–2, 747
 variable 144, 146, 150, 747
 Cramer's rule 575–88
 meaning of term 576, 583, 742
 critical points 337, 353, 747
see also stationary points
 cross-multiplication 34
 cross-price elasticity of demand 415,
 416, 425, 742
 cubic equations 191, 340–2
 curves
 concave 293, 294, 741
 convex 293, 294, 742
 indifference curves 419–22, 425,
 459, 744
 isocost curves 458–9, 466, 744
 isoquants 423, 424, 425, 458, 459, 744
 L-shaped curves 145, 146, 150, 744
 maximum (local) point 337–8,
 353, 745
 minimum (local) point 338, 353, 745
 sketching from function formulae
 132–4
 sketching from tables of numbers
 130, 131–2, 145, 183, 280–1
 slopes 278–9, 285
 tangents to 278–9, 285, 299, 384–6,
 483, 484, 747
 U-shaped curves 130–8, 280–1, 747
see also graphs
 data, nominal distinguished from real
 214–15
 data points
 extraction of formulae from 182–5
 straight lines drawn from 184
 decision variable 619, 626, 742
 decreasing functions 69, 80, 742
 decreasing returns to scale, production
 function with 162, 163, 170, 742
 definite integrals 510, 520, 742
 definite integration 510–24
 meaning of term 500, 505, 742
 degree of homogeneity 162–3, 170, 742
 degree of polynomial 191, 193, 742
 demand
 cross-price elasticity of 415, 416,
 425, 742
 elastic 322, 323, 334, 742
 income elasticity of 415, 416, 425, 743
 inelastic 322, 323, 334, 744
 price elasticity of 322–8, 330–3, 334,
 362–3, 414–15, 416, 426, 746
 total demand for money 101
 unit elasticity of 322, 323, 334, 747
see also elasticity of demand; supply
 and demand analysis
 demand curves 69–70, 72, 73, 74, 301,
 302, 331, 332, 333
 demand for money 101
 precautionary demand 101, 105, 746
 speculative demand 101, 105, 257, 747
 transactions demand 101, 105, 747
 demand functions 68–9, 80, 742
 consumer's surplus 513–14, 520
 quadratic 136–8
 denominators 36, 38, 742
 dependent variables 68, 80, 401, 410, 742
 derivatives
 first-order 292, 293, 294, 338, 743
 first-order partial 402–4, 405, 406,
 446, 449, 452, 471, 472
 of functions 280, 285
 gradient of tangent to curve 280, 293,
 384–5, 742
 natural logarithms 369–83
 partial 402–7, 411, 446, 449, 450, 452,
 471, 472, 483–4, 746
 second-order 292–3, 294, 338–9, 746
 second-order partial 404–6, 411, 446,
 449, 450, 452, 472, 483, 746
see also differential equations;
 differentiation; marginal
 functions
 derived functions 280, 285, 742
 determinants of matrices 557–8, 571, 742
 calculation of 565–7, 575–7
 difference equations 644–58, 675
 complementary functions 645–6, 648,
 649, 651, 653, 655, 741
 equilibrium values 648–9, 655, 743
 exploding time paths 648
 general solutions 645, 646, 648, 651,
 653, 655, 743
 graphical interpretation of solutions
 647–8, 649
 initial conditions 645, 655, 744
 linear models 644–54
 meaning of term 644, 655, 742
 national income determination 650–2
 non-linear problems 655
 oscillatory time paths 650, 653, 655
 particular solutions 645–6, 648, 649,
 651, 653, 655, 746
 stable models 650, 651, 652, 653, 655
 supply and demand analysis 652–4
 uniformly converging sequences/time
 paths 648–9, 650, 651, 652, 655
 uniformly diverging sequences/time
 paths 648, 650, 655
 unstable models 650, 655
 difference of two squares formula 18,
 19, 742
 difference rule of differentiation 289–90,
 291, 499
 differential calculus 275–386
see also differentiation
 differential equations 659–73
 adjustment coefficients 665, 669, 741
 arbitrary constants 659, 669, 741
 complementary functions 661–2, 663,
 664, 666, 668, 669, 741
 equilibrium values 663, 664, 669, 743
 general solutions 659, 662, 663, 664,
 666, 668, 669, 743
 graphical interpretations 663, 664
 initial conditions 659, 669, 744
 meaning of term 644, 659, 669, 742
 national income determination 665–7
 particular solutions 661, 662, 666,
 668, 669, 746
 stable models 664–5, 666, 667, 669
 supply and demand analysis 667–9
 unstable models 665
 differential pricing 359–63, 451–3
 differentials 404, 410, 742

- differentiation 275–386
 algebraic fractions 317–19
 consumption 307, 308
 exponential functions 369–80, 497
 implicit differentiation 409–10, 411, 421, 743
 meaning of term 282, 285, 742
 natural logarithms 372, 496–7
 optimisation of economic functions and 337–68
 partial differentiation 399–491
 power functions 282–4
 production functions 305–7, 309
 rules
 chain rule 312–14, 315, 316, 317, 372, 373, 377, 431, 437, 504–5
 constant rule 287–8, 291, 498
 difference rule 289–90, 291, 498
 product rule 315–17, 372, 373, 376
 quotient rule 318–19, 363, 372, 373
 sum rule 288–9, 291, 498
 savings 307, 308
 total cost(s) 303–4, 309
 total revenue 298–302, 309
 see also derivatives; partial differentiation
- diminishing marginal productivity, law of 306–7, 309, 744
- diminishing marginal utility, law of 419, 425, 460, 744
- diminishing returns, law of 306–7, 309, 744
- discontinuous functions 191, 193, 742
- discount rates 247, 259, 742
- discounting 247, 259, 518, 742
- discrete compounding of interest 223–7, 247
 discount formula 223, 224, 247, 248
- discriminants 128, 138, 742
- discrimination, price 359–63, 451–3
- disposable income 98, 105, 578–80, 742
 in three-sector macroeconomic model 578–80
- distributive law 14–16, 19, 548, 742
 applied in reverse 16–18
 in matrix algebra 548
- diverges uniformly, meaning of term 648, 650, 655
- division
 algebraic fractions 26–7
 exponential forms 157, 158–9
 fractions 26–7
 matrices 537
 negative numbers 9, 136
 by scale factors 207, 208
 by zero 31, 111, 136, 496
- domain 111, 112, 742
- dynamics 643–74
 difference equations 644–58
 differential equations 659–73
 meaning of term 431, 439, 742
- e 175, 177–8, 660
 continuing compounding of interest and 227–8
 differential equations 660
 logarithms to base e 180–5
 see also natural logarithms
- economic functions, optimisation of 337–68
- economic order quantity (EOQ) 366, 367, 742
- economies of scale 146
- elastic demand 322, 323, 334, 742
- elasticity 322–36
 arc elasticity 324–5, 329, 334, 741
 marginal revenue and 330–1
 point elasticity 325, 329, 334
- elasticity of demand 322–8
 cross-price 415, 416, 425, 742
 income 415, 416, 425, 743
 marginal revenue and 330–1, 362
 partial differentiation of 414–17
 price 322–8, 330–3, 334, 362–3, 414–15, 416, 426, 746
 quadratic equations and 327–8
 unit 322, 323, 334, 747
- elasticity of supply, price elasticity of supply 328–30, 334, 746
- elements of matrices 536, 551, 742
- elimination method 55–64, 559, 570
 meaning of term 55, 65, 742
- endogenous variables 72, 80, 742
- entries of matrices 536, 551
- equations
 algebraic 31–7, 38, 742
 cubic 191, 340–2
 difference equations 644–58, 742
 differential equations 659–73, 742
 linear 7–112, 744
 mathematical operations applied to 31–4
 non-linear 123–200
 quadratic 31, 124–41
 solving 31
 structural 430, 439, 578, 747
 see also difference equations; differential equations; linear equations; non-linear equations; quadratic equations; simultaneous linear equations
- equilibrium
 market 67, 74–5, 80, 136, 515, 743
 money market 101
 stable (difference and differential equations) 655, 747
 unstable (difference and differential equations) 655, 747
- equilibrium consumption 560
- equilibrium income 97, 560
- equilibrium price 67, 74, 436–7
 integration and 515–16
 matrix-based calculations 559–60, 569–70
- equilibrium quantity 67, 74, 437–8, 515–16
- equilibrium values
 difference equations 648–9, 655, 743
 differential equations 663, 664, 669, 743
- ‘equivalence’ symbol 111, 112
- equivalent fractions 36–7, 38, 743
- Euler’s theorem 425, 743
- exogenous variables 72, 74, 80, 743
- ‘expanding the brackets’ 14–16, 17–18
- exploding time paths 648
- exponential forms 153, 164, 170, 743
 see also power(s)
- exponential functions 177–80, 185, 743
 compound interest and 228
 differentiation of 369–80, 497
 graphical representation 175–6, 369–70
 integration of 497
- exponents 165, 170, 175, 743
 negative 154, 160, 165, 175, 176
 see also power(s)
- expressions, algebraic 11–14
- extrema 337, 353, 747
 see also stationary points
- factor of an expression 38, 743
- factorisation 16–17, 18, 19, 743
 quadratic equations 129–30
- factors of production 74, 93, 105, 161, 170, 743
- feasible regions (in linear programming) 604–10, 611
 applications 620, 622, 623–4, 625
 meaning of term 614, 743
 unbounded 613, 614, 747
- finance 203–65
 compound interest 222–35
 geometric series 236
 investment appraisal 247–62
 percentages 204–21
- firms (in national economy model) 93, 96, 97, 430
- first-order derivatives 292, 293, 294, 338, 743
- first-order partial derivatives 402–4, 405, 406, 446, 449, 452, 471, 472
- fixed costs (FC) 144, 150, 743
- flow charts 86–8, 91, 743
 reverse 86, 87, 88, 91, 746
- ‘for all’ symbol 111
- foreign trade, in macroeconomic model 581–3
- formulae
 extraction from data points 182–5
 sketching curves from 132–4
 transposition of 84–92, 747
- fractional indices/powers 155–6, 284
- fractions
 addition of 27–9
 algebraic fractions 24–31, 38, 317–19, 741

- division of 26–7
 - equivalent fractions 36–7, 38, 743
 - multiplication of 26–7
 - in simultaneous linear equations 55
 - subtraction of 27–9
 - functions 67–8
 - consumption function 93, 94, 95, 97, 105, 502–3, 741
 - continuous 191, 192, 193, 741
 - decreasing 69, 80, 742
 - defined piecewise 191, 193
 - derivatives of 280, 285
 - derived functions 280, 285, 742
 - discontinuous 191, 193, 742
 - economic functions, optimisation of 337–68
 - exponential functions 177–80, 185, 743
 - of functions 312
 - homogeneous 162, 170, 425, 743
 - increasing 73, 80, 743
 - inverse 68, 80, 744
 - Lagrangian 471, 479, 744
 - marginal 298–311
 - meaning of term 67–8, 80, 743
 - objective functions 457, 466, 607, 608, 609, 610, 611, 613, 625
 - power functions
 - differentiation of 282–4
 - integration of 496–7
 - production functions 161–3, 170, 746
 - quadratic functions 124–41
 - savings function 94–5, 96
 - of several variables 400–13
 - partial differentiation of 402–7
 - pictorial representation 402
 - simple functions, direct way of
 - integrating 496
 - supply functions 73–4, 80, 747
 - of two variables 400–1, 410, 743
 - see also* complementary functions; demand functions; production functions
 - future value (with compound interest) 223, 232, 743
 - continuous-compounding calculation 227
 - general solutions
 - difference equations 645, 647, 648, 651, 653, 655, 743
 - differential equations 659, 662, 663, 664, 666, 668, 669, 743
 - geometric progression 236, 244, 743
 - geometric ratio 236, 244, 743
 - geometric series 236
 - compound interest and 236
 - loan repayments 240–2
 - meaning of term 237, 244, 743
 - non-renewable commodities 242–4
 - savings plans 238–40
- Gini coefficient 519–20, 520, 743
 - GNP (gross national product), annual
 - growth 230–1
 - good(s)
 - complementary 72, 77, 80, 741
 - inferior 73, 80, 415, 744
 - normal 73, 80, 745
 - substitutable 72, 77, 80, 747
 - superior 415, 426, 747
 - government bonds 257–9
 - government expenditure 98, 105, 433, 743
 - government expenditure multiplier 434, 435
 - gradients
 - of curves 278–9, 285, 743
 - of straight lines 49, 50, 51, 52, 276–8, 279, 285, 743
 - graphs
 - area determined by integration 509–15, 525–6
 - average cost 145, 146, 147
 - axes 42, 52
 - constrained optimisation 458–60
 - continuous functions 191
 - coordinates 42, 52, 742
 - cubic functions 341–2
 - difference equations 647–8, 649
 - differential equations 663, 664
 - discontinuous functions 191
 - exponential functions 175–6, 369–70
 - feasible regions 604–6, 607–8, 609–10, 611, 613, 614, 620, 624, 625
 - functions of several variables 402
 - gradients 49, 50, 51, 52, 276–8, 279, 285, 743
 - indifference curves 419–22, 425
 - inequalities 600–3
 - intercepts 46, 49, 50, 51, 52, 744
 - intersection points of two curves 138
 - intersection points of two lines 47–8
 - isocost curves 458–9, 466, 744
 - isoquants 423, 424, 425, 458, 459, 744
 - L-shaped curves 145, 146, 150, 744
 - linear equations 42–54, 130
 - linear programming 604–17, 620, 624, 625
 - modulus function 386
 - origin 42, 52, 745
 - quadratic functions 130–8
 - sketching curves from formula 132–4
 - sketching curves from table of values 130, 131–2, 145, 183, 280–1
 - sketching lines from equations 44–52
 - sketching lines from table of values 184
 - slope–intercept approach 50–1
 - slopes 49, 50, 51, 52, 276–8, 279, 285
 - stationary points 337–43
 - tangents to 278–9, 285
 - three-dimensional 402
 - total cost function 145, 146–7
 - total revenue functions 142–3, 146–7, 299
 - U-shaped curves 130–8, 280–1
 - unbounded regions 525–6
 - gross national product (GNP), annual
 - growth 230–1
 - growth
 - limited 179, 185, 744
 - unlimited 182, 185, 747
 - holding costs 364–6
 - homogeneity, degree of 162–3, 170, 742
 - homogeneous functions 162, 170, 425, 743
 - partial differentiation of 425
 - households (in national economy model) 93, 96, 430
 - hyperbolas, rectangular 145, 146, 150, 746
 - identities 31, 38, 743
 - identity matrices 556, 563, 571, 743
 - implicit differentiation 409–10, 411, 421, 743
 - ‘implies’ symbol 111, 112
 - imports *see* marginal propensity to import multiplier
 - improper integral 525, 526, 743
 - income 93
 - disposable 98, 105, 578–80, 742
 - see also* equilibrium income; national income
 - income constraints 457
 - income elasticity of demand 415, 416, 425, 743
 - increasing functions 73, 80, 743
 - increasing returns to scale, production
 - functions with 162, 163, 170, 743
 - indefinite integrals 500–1, 505
 - indefinite integration 494–508
 - meaning of term 505, 743
 - independent variables 68, 80, 401, 411, 744
 - index
 - meaning of term 153, 170, 744
 - see also* indices
 - index notation 153–6, 169
 - index numbers 210–14, 216, 744
 - Laspeyres index 213, 216, 744
 - Paasche index 214, 216, 745
 - percentages and 212–13
 - indices
 - negative 154, 160, 165, 175, 176
 - rules of 157–63, 169
 - see also* power(s)
 - indifference curves 419–22, 425, 459, 744
 - indifference map 419, 425, 459, 744
 - inelastic demand 322, 323, 334, 744
 - inequalities 35–7
 - linear 600–3
 - sign diagram 135–6
 - simplification of 37
 - see also* linear programming

- inferior good(s) 73, 80, 415, 744
- inflation 214–16, 744
- inflection points 338, 353, 747
- initial conditions
 difference equations 645, 655, 744
 differential equations 659, 669, 744
- integer programming 625, 626, 744
- integrals 495, 505, 744
 definite 510, 520, 742
- integration 493–532, 744
 constants of 496, 505, 511, 741
 definite integration 510–24
 direct way for simple functions 496
 exponential functions 497
 indefinite integration 494–508
 limits 510, 520, 744
 meaning of term 494, 505
 power functions 496–7
 rules 498
- intercepts of graphs 46, 49, 50, 51, 52, 744
- interest
 compound 222–35, 741
 interest on 222, 227
 simple 222, 232, 746
 see also compound interest
- interest rates
 discount rates 247, 259, 742
 in national income determination 100–4
 speculative demand for money and 101, 257
- internal rate of return (IRR) 249–50, 251–2, 255–7, 259, 744
 limitations 250, 252
- intersection points
 of two curves 138
 of two lines 47–8
- intervals 110–11, 112, 744
 closed 110, 112, 741
 open 110, 112, 745
- inverse functions 68, 80, 744
- inverse matrix 537, 557, 571, 744
 construction of 567–9
 linear equations solved using 558–61, 569–70
- inverses (mathematical operations) 494, 505, 744
- inversion of matrices 556–74
- investment 96, 105, 744
 net 516, 519, 745
- investment appraisal 247–62
 annuities 252–3, 259, 518, 741
 government bonds 257–9
 internal rate of return (IRR) 249–50, 251–2, 255–7, 259, 744
 net present value (NPV) 248, 249, 250, 251, 253–4, 259, 745
 present values 248
- investment flow 516–17
- investment multipliers 431, 432, 439, 562, 744
- IRR *see* internal rate of return
- IS schedule 101, 102, 103, 105, 744
- isocost curves 458–9, 466, 744
- isoquants 423, 424, 425, 458, 459, 744
- L-shaped curves 145, 146, 150, 744
- labour 161, 170, 305, 744
 average product 345, 346, 353, 741
 optimisation of 345–6, 363–4
 marginal product 305–6, 309, 345, 346, 363–4, 423, 426, 745
- labour productivity 345, 353, 744
- Lagrange multipliers 470–82
 meaning of term 471, 479, 744
- Lagrangian function 471, 479, 744
- Laspeyres index 213, 216, 744
- law of diminishing marginal productivity 306–7, 309, 744
- law of diminishing marginal utility 419, 425, 460, 744
- law of diminishing returns 306–7, 309, 744
- laws
 associative law 548
 commutative law 548, 549
 distributive law 14–16, 19, 548, 742
- like terms 13, 19, 744
- limited growth 179, 185, 744
- limits
 in differentiation 384–6
 exponential 177–8
 of functions 192–3
 of integration 510, 520, 744
 sigma notation 264, 265
- linear demand equation 69, 301, 302
- linear difference equations 644–54
- linear equations 7–112
 algebra 8–41, 55–66
 coefficients 44, 52
 graphs representing 42–54, 130
 mathematical operations applied to 31–4
 matrix-based solutions 558–61
 meaning of term 44, 52, 744
 national income determination using 93–109
 sketching lines from 44–52
 supply and demand analysis 67–83
 transposition of formulae 84–92
 see also simultaneous linear equations
- linear inequalities, graphical representation 600–3
- linear programming 599–640
 applications 618–31
 graphical solutions 600–17
 n variables 632
 problem formulation 600 618–26
- LM schedule 101, 102, 103, 104, 105, 744
- loan repayments, geometric series 240–2
- local maxima and minima 337–8, 353, 745
- logarithms 163–9, 170, 744
 compound interest calculations 225
 rules 165–6, 169, 180–1, 374
 see also natural logarithms
- Lorenz curve 518–20, 744
- lower limit 264, 265, 745
- macroeconomics
 comparative statics 430–6
 difference equations 650–2
 differential equations 665–7
 matrices used to solve linear equations 560–2
 Cramer's rule used 578–83
 national income determination 93–109
 difference equations used 650–2
 differential equations used 665–7
 percentages 210–16
 three-sector model 433–5, 578–80
 two-sector model 93–6, 100–1, 430–2, 560–2, 650–2, 665–7
- marginal cost(s) 303–4, 309, 745
- integration of 501–2, 503
- optimisation of economic functions 348, 356, 357–8
- marginal functions 298–311
- integration of 501–3
- marginal product 459
- marginal product of capital 423, 425, 745
- marginal product of labour 305–6, 309, 345, 346, 363–4, 423, 426, 745
- marginal productivity, diminishing, law of 306–7, 309, 744
- marginal propensity to consume (MPC) 93, 95, 105, 307–8, 309, 745
 in dynamic conditions 650, 652, 667
 integration of 502
- marginal propensity to consume multiplier 431, 432, 439, 745
- marginal propensity to import 435
- marginal propensity to import multiplier 435
- marginal propensity to save (MPS) 95, 105, 307–8, 309, 745
 integration of 503
- marginal rate of commodity substitution (MRCS) 420–2, 426, 745
- marginal rate of technical substitution (MRTS) 423–4, 426, 745
- marginal revenue 298–301, 302, 309, 745
 demand elasticity and 330–1, 362
 integration of 502, 503
 optimisation of economic functions 348, 356, 357–8

- marginal utility 417–19, 426, 460, 745
diminishing, law of 419, 425, 460, 744
- market equilibrium 67, 74–5, 80, 136, 515, 743
producer's surplus and 515–16
quadratic functions and 136–8
- 'market forces' 74
- market saturation level 179
- mathematical notation 111
- mathematical operations
applying to equations 31–4
inverses 494, 505, 744
- matrices 535–96
addition of 539–41
adjoint matrices 567, 568
adjugate matrices 567, 568
algebra 548, 556
associative law 548
basic operations 536–55
cofactors of elements 563–7, 571, 741
column vectors 539, 551, 741
columns 536
commutative law 548, 549
Cramer's rule 575–88
determinants 557–8, 571, 742
calculation of 565–7, 575–7
distributive law 548
division of 537
elements (entries) 536, 551, 742
identity matrices 556, 563, 571, 743
inverse matrix 537, 557, 571, 744
construction of 567–9
linear equations solved using 558–61, 569–70
inversion of 556–74
linear equations solved using 558–61
meaning of term 536, 551
multiplication of 543–51
general 546–8
row vectors by column vectors 543–6
by scalar quantities 542–3
'non-property' 549, 551
non-singular matrices 557, 571, 745
notation 536–7, 550
linear programming 632
orders 536, 551, 745
row vectors 539, 551, 746
rows 536
sigma notation 589
simultaneous linear equations solved using 550–1
singular matrices 557, 571, 746
square matrices 556, 571, 747
subtraction of 539–41
transposition of 538–9, 551, 747
zero matrices 541, 551, 747
- matrix, meaning of term 536, 551, 745
- maxima
curves 337–8, 353, 745
U-shaped curves 134
- maximisation *see* optimisation
- maximum (local) point 337–8, 353, 745
- maximum point (of function of two variables) 444, 445, 454, 745
- maximum profit, calculation of 147–9, 347–8, 444, 448–53
- method of substitution 460–2, 466, 745
- microeconomics 67
comparative statics 436–8
difference equations 652–4
differential equations 667–9
market equilibrium 67–83, 136–8
matrices used to solve linear equations 560–2
profit calculations 142–52
quadratic functions 136–8
supply and demand analysis 67–83, 436, 652–4, 667–9
- minima
curves 338, 353, 745
U-shaped curve 130
- minimisation *see* optimisation
- minimum (local) point 338, 353, 745
- minimum point (of function of two variables) 444–5, 454, 745
- minor 563, 571, 745
- modelling 69, 80, 745
- modulus 110, 112, 745
graph 386
tangent 386
- money
precautionary demand for 101, 105, 746
speculative demand for 101, 105, 257, 747
transactions demand for 101, 105, 747
- money market equilibrium 101
- money supply 101, 105, 745
- monopolists 301–2, 309, 745
in constrained optimisation problems 473–5
in indefinite integration 501, 505
in unconstrained optimisation problem(s) 450
- MPC *see* marginal propensity to consume
- MPS *see* marginal propensity to save
- MRCs *see* marginal rate of commodity substitution
- MRTS *see* marginal rate of technical substitution
- multiplication
algebraic fractions 26–7
brackets 14–16, 17–18
cross-multiplication 34
exponential forms 157, 158–9
fractions 26–7
matrix
general 546–8
row vectors by column vectors 543–6
- by scalar quantities 542–3
sigma notation 589
negative numbers 9
by scale factors 207, 208
of successive scale factors 209–10
- multipliers 431, 745
- autonomous consumption 431, 439, 562, 741
- autonomous export 435
- autonomous taxation 434
- balanced budget 434, 439, 741
- government expenditure 434, 435
- investment 431, 432, 439, 562, 744
- Lagrange 470–82
- marginal propensity to consume 431, 432, 439, 745
- marginal propensity to import 435
- national economy models 96–104
three-sector model 433–5, 578–80
two-sector model 93–6, 100–1, 430–2, 560–2, 650–2, 665–7
- national income 93, 105, 307, 430, 745
marginal functions and 307–8
- national income determination
difference equations 650–2
differential equations 665–7
linear equations 93–109
- natural logarithms 180–5, 745
- continuous compounding of interest and 228
- derivatives 369–83
differentiation of 372, 496–7
integration and 497, 499
rules 374
- negative exponents/indices/powers 154, 160, 165, 175, 176
- negative numbers 9–11
addition of 10
division of 9, 136
division of inequality by 36
multiplication of 9
multiplication of inequality by 36
square roots 111, 128
subtraction of 10–11
- net investment 516, 520, 745
- net present value (NPV) 248, 249, 250, 251, 253–4, 259, 745
- nominal data 214–15, 216, 745
- non-linear equations 123–200
difference equations 655
quadratic equations 31, 124–41
revenue, cost and profit 142–52
simultaneous 446–7, 449, 452
- non-negativity constraints 607, 609, 612
applications 619, 620, 621, 623
general linear programming problem for n variables 632
meaning of term 614, 745

- non-renewable commodities, geometric series calculations 242–4
- non-singular matrices 557, 571, 745
- normal good(s) 73, 80, 745
- NPV *see* net present value
- number line 10, 35, 38, 110, 135–6, 745
- numbers
 - index numbers 210–14, 216, 744
 - negative numbers 9–11
- numerators 36, 38, 745

- objective functions 457, 466, 607, 608, 609, 610, 611, 613
 - applications 619, 621, 622, 624, 625
 - constrained optimisation of 457–69
 - linear programming used 621–5
 - general linear programming problems for n variables 632
 - meaning of term 614, 745
- one-commodity market model 67–77, 436
 - in dynamic conditions 652–4, 667–9
- open interval 110, 112, 745
- operations *see* mathematical operations
- optimisation
 - average cost 349–50, 364
 - average product of labour 345–6, 363–4
 - constrained 457–69, 606–14
 - economic functions 337–68
 - meaning of term 343, 353, 745
 - production functions 343–6, 457–9, 462–6
 - profit 347–8, 356–62
 - tax revenue 351–2
 - total revenue 346–7
 - unconstrained 443–56
- order of matrix 536, 551, 745
- ordering costs 364–6
- origin of graph 42, 52, 745
- oscillatory time paths, difference equations 650, 653, 655
- output *see production entries*
- output constraints 464–6
- output growth 231
- own price elasticity of demand 414–15

- Paasche index 214, 216, 745
- parabolas 130–8, 746
 - turning points 337–9
- parameters 69, 80, 746
- partial derivatives 402–7, 411, 446, 449, 450, 452, 471, 472, 483–4, 746
 - first-order 402–4, 405, 406, 446, 449, 452, 471, 472
 - second-order 404–6, 411, 446, 449, 450, 452, 472, 483, 746
- partial differentiation 399–91
 - comparative statics 430–42
 - constrained optimisation 457–69
 - elasticity of demand 414–17
 - functions of several variables 402–7
 - Lagrange multipliers 470–82
 - production functions 414–29
 - unconstrained optimisation 443–56
 - utility functions 417–22
- particular solutions (PS)
 - difference equations 645, 646–7, 649, 651, 653, 655, 746
 - differential equations 661, 662, 666, 668, 669, 746
- percentages 204–21
 - calculations using 205–6
 - index numbers and 210–14
 - inflation and 214–15
 - interest rate 229
 - scale factors and 207–10, 215
- perfect competition 69, 302–3, 309, 448, 515, 746
- point elasticity 325, 329, 334, 746
- points of inflection 338, 353, 747
- points of intersection
 - of two curves 138
 - of two lines 47–8
- polynomial expressions 191, 193, 746
- power functions
 - differentiation of 282–4
 - integration of 496–7
- power(s) 153, 170, 746
 - fractional 155–6, 284
 - negative 154, 160, 165, 175, 176, 284
 - power of 157–8
 - product of two numbers 158
 - rules of 157–63, 169
 - zero 155, 169
- precautionary demand for money 101, 105, 746
- present value 247–8, 258, 259, 518, 746
 - investment appraisal 248
 - see also* net present value
- price *see* equilibrium price; shadow price
- price discrimination 359–63, 451–3
- price elasticity of demand 322–8, 334, 414–15, 416, 426, 746
 - marginal revenue and 330–1, 362
- price elasticity of supply 328–30, 334, 746
- primitives 495, 505, 746
 - see also* integrals
- principal 11, 223, 232, 746
 - see also* future value; present value
- problem formulation in linear programming 600, 618–26
- producer's surplus 514–16, 520, 746
- product rule of differentiation 315
 - applications 315–17, 372, 373, 376
- production, factors of 74, 93, 105, 161, 170, 743
- production functions 161–3, 170, 746
 - with constant returns to scale 162, 163, 170, 741
 - constrained optimisation of 457–9, 462–6
 - with decreasing returns to scale 162, 163, 170, 742
 - differentiation of 305–7, 309
 - homogeneous 162, 170
 - with increasing returns to scale 162, 163, 170, 743
 - isoquants 423, 424, 425
 - optimisation of 343–6, 457–9, 462–6
 - partial differentiation of 423–5
 - see also* Cobb–Douglas production functions
 - productivity, labour productivity 345, 353
 - profit
 - maximum 147–9
 - meaning of term 142, 150, 746
 - optimisation of 347–8, 444, 448–53
 - linear programming and 618–22
 - progression
 - arithmetic 237, 244, 741
 - geometric 236, 244, 743
 - pure competition *see* perfect competition

 - quadratic equations 31, 124–41
 - demand elasticity and 327–8
 - factorisation of 129–30
 - solving 125–30, 339–40
 - quadratic functions 124–41
 - demand functions 136–8
 - graphs representing 130–8
 - meaning of term 138, 746
 - supply functions 136–8
 - quantity, equilibrium 67, 74, 437–8, 515–16
 - quotient rule of differentiation 318
 - applications 318–19, 363, 372, 373

 - range 111, 112, 746
 - real data 214–15, 216, 746
 - reciprocals 160
 - differentiation of natural logs 372, 377
 - integration of 496, 497, 499
 - negative powers evaluated as 154, 160, 165, 175, 176, 284
 - rectangular hyperbola curves 145, 146, 150, 746
 - recurrence relation 644, 655, 746
 - see also* difference equations
 - reduced form (macroeconomic model) 431, 439, 746
 - relative maxima and minima 337–8
 - revenue
 - average 302, 309, 741
 - continuous streams 518
 - marginal 298–301, 302, 309, 745
 - optimisation of economic functions 348, 356, 357–8
 - total 142–4, 150, 322, 747
 - reverse flow charts 86, 87, 88, 91, 746
 - roots
 - fractional powers as 155–6, 284
 - see also* square roots

- row vectors in matrices 539, 551, 746
multiplication by column vectors 543–6
- rows in matrices 536, 538
- rules
Cramer's rule 575–88
differentiation 287–97
indices/exponents/powers 157–63, 169
integration 498
logarithms 165–6, 169 180–1, 374
natural logarithms 374
powers 157–63, 169
- saddle points 444, 445, 454, 746
- savings 93
autonomous 95, 105, 741
differentiation of 307, 308
marginal propensity to save 95, 105, 307–8, 309, 503, 745
- savings function 94–5, 96
- savings plan, geometric series 238–40
- scalar multiplication of matrices 542–3
- scalar quantities, distinguished from vector quantities 539
- scale economies 146
- scale factors 207–10, 215, 216, 746
compound-interest calculations and 223
division by 207, 208
multiplication by 207, 208
multiplication of successive 209–10
- second-order derivatives 292–3, 294, 338–9, 746
stationary points and 338–9
- second-order partial derivatives 404–6, 411, 446, 449, 450, 452, 472, 483, 746
- shadow price 622, 626, 746
- sigma notation 264–5
linear programming 632
matrices 589
- sign diagram 135–6
- simple interest 222, 232, 746
compared with compound interest 222
- simplex algorithm 618
- simultaneous linear equations 52, 611, 746
algebraic solutions 55–66
graphical solutions 47–8, 52
infinitely many solutions 59–60
linear programming 621
matrix-based solution 550–1
no solution 58–9
with three unknowns 61–4
- simultaneous linear inequalities 604, 611–14
- simultaneous non-linear equations, solving 446–7, 449, 452
- singular matrices 557, 571, 746
- sinking fund 238, 242, 746
- slope–intercept approach to solving linear equations 50–1
- slopes
of curves 278–9, 285
of line 746
slopes of 292, 293
of straight lines 49, 50, 51, 52, 276–7, 279, 285
- small increments formula 407–9, 411, 747
- speculative demand for money 101, 105, 257, 747
interest rates and 101, 257
- spreadsheets, and difference equations 655
- square brackets for matrices 536
- square function (x^2)
curves 130, 280–1
differentiation of 282
- square matrices 556, 571, 747
- square roots 125, 138, 747
of negative numbers 111, 128
- stable equilibrium (difference and differential equations) 655, 669, 747
- stable models
difference equations 650, 651, 652, 653, 655
differential equations 664–5, 666, 667, 668, 669
- statics
comparative 430–42
meaning of term 431, 439, 747
- stationary point of inflection 338, 353, 747
- stationary points 337–43, 353, 747
constrained optimisation 444–8, 461, 462
profit functions 356–7
unconstrained optimisation 444
- stock holding problem(s) 364–6
- straight lines
gradients/slopes 49, 50, 51, 52, 276–8, 279, 285, 743
linear programming problems, graphical solution 607–8, 611–12, 613, 614
sketching from data points 184
sketching from equations 44–52
see also graphs
- structural equations 430, 439, 578, 747
reduced form 431, 560
- substitutable good(s) 72, 77, 80, 747
- substitution, method of 460–2, 466, 745
- subtraction
fractions 27–9
matrices 539–41
negative numbers 10–11
- sum rule of differentiation 288–9, 291, 498
- superior good(s) 415, 426, 747
- supply
money supply 101, 105, 745
price elasticity of 328–30, 334, 746
- supply analysis *see* supply and demand analysis
- supply and demand analysis
difference equations 548–50
differential equations 667–9
linear equations 67–83
one-commodity market model 652–4, 667–9
three-commodity market model 79
two-commodity market model 77–9
- supply curves 73–4
- supply functions 73–4, 80, 747
producer's surplus 514–16, 520
quadratic 136–8
- surplus
consumer's 513–14, 520, 741
producer's 514–16, 520, 746
- tables of function values
sketching curves from 130, 131–2, 145, 183, 280–1
sketching lines from 184
- tangents to curves 278–9, 285, 299, 747
partial derivatives 483, 484
slopes 384–5
- taxation 98–9, 105, 747
autonomous taxation multiplier 434
optimisation of 351–2
supply and demand analysis and 75–7
in three-sector macroeconomic model 433, 560–2
- technical substitution, marginal rate of 423–4, 426, 745
- 'there exists' symbol 111
- 'therefore' symbol 111
- three-commodity market model, supply and demand analysis 79
- three-dimensional graphs 402
- three-sector [national economy] model 433–5, 578–80
- time paths
exploding 648
oscillatory 650, 653, 654
uniformly convergent 648–9, 747
uniformly divergent 648, 747
- time series 210, 212, 216, 747
- total cost (TC) 142, 144, 145, 146, 150, 359, 501–2, 747
differentiation of 303–4, 309
graphs 145, 146–7
integration of marginal costs 501–2
profit optimisation and 359
- total demand for money 101
- total revenue (TR) 142–4, 150, 322, 747
differentiation of 298–302, 309
graphs 142–3, 146–7, 299
integration of marginal revenue 502
non-linear equations 144
optimisation of economic functions 346–7
profit optimisation and 359–60

- total variable cost (TVC) 144
- trading nations, macroeconomic models covering 581–3
- transactions demand for money 101, 105, 747
- transpose of matrix 538–9, 551, 747
- transposition
 - of formulae 84–92, 747
 - of matrices 538–9, 747
- turning points 337, 353
 - see also* stationary points
- two-commodity market model, supply and demand analysis 77–9
- two-sector [national economy] model 93–6, 100–1, 430–2, 560–2
 - difference equations 650–2
 - differential equations 665–7
- U-shaped curves 130–8, 747
 - sketching from function formulae 132–4
 - sketching from table of function values 130, 131–2, 280–1
- unbounded (feasible) region 613, 614, 747
- unbounded (graphical) regions, area of 525–6
- unconstrained optimisation 443–56
- uniformly convergent sequences/time paths 648–9, 650, 651, 652, 655, 747
- uniformly divergent sequences/time paths 648, 650, 655, 747
- unit elasticity of demand 322, 323, 334, 747
- unlimited growth 182, 185, 747
- unstable equilibrium (difference equations) 655, 747
- unstable models
 - difference equations 650, 655
 - differential equations 665
- upper limit 264, 265, 747
- utility 417–18, 426, 747
 - marginal 417–19, 426, 460, 745
- utility functions
 - constrained optimisation of 459–62
 - partial differentiation of 417–22
 - unconstrained optimisation of 443
- variable costs (VC) 144, 146, 150, 747
- variables
 - decision 619, 626, 742
 - dependent 68, 80, 401, 410, 742
 - endogenous 72, 80, 742
 - exogenous 72, 74, 80, 743
 - functions of several variables 400–1
 - partial differentiation of 402–7
 - pictorial representation 402
 - functions of two variables 400–1, 410, 743
 - independent 68, 80, 401, 411, 744
- vectors
 - distinguished from scalars 539
 - see also* column vectors in matrices; row vectors in matrices
- wages optimisation, linear programming 622–5
- x axis of graph 42, 52, 747
- y axis of graph 42, 52, 747
- Young's theorem 406
- zero
 - division by 31, 111, 136, 496
 - as index/power 155, 169
- zero matrix 541, 551, 747