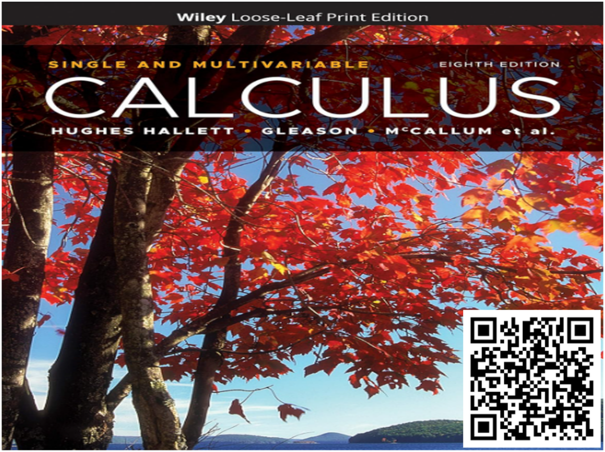
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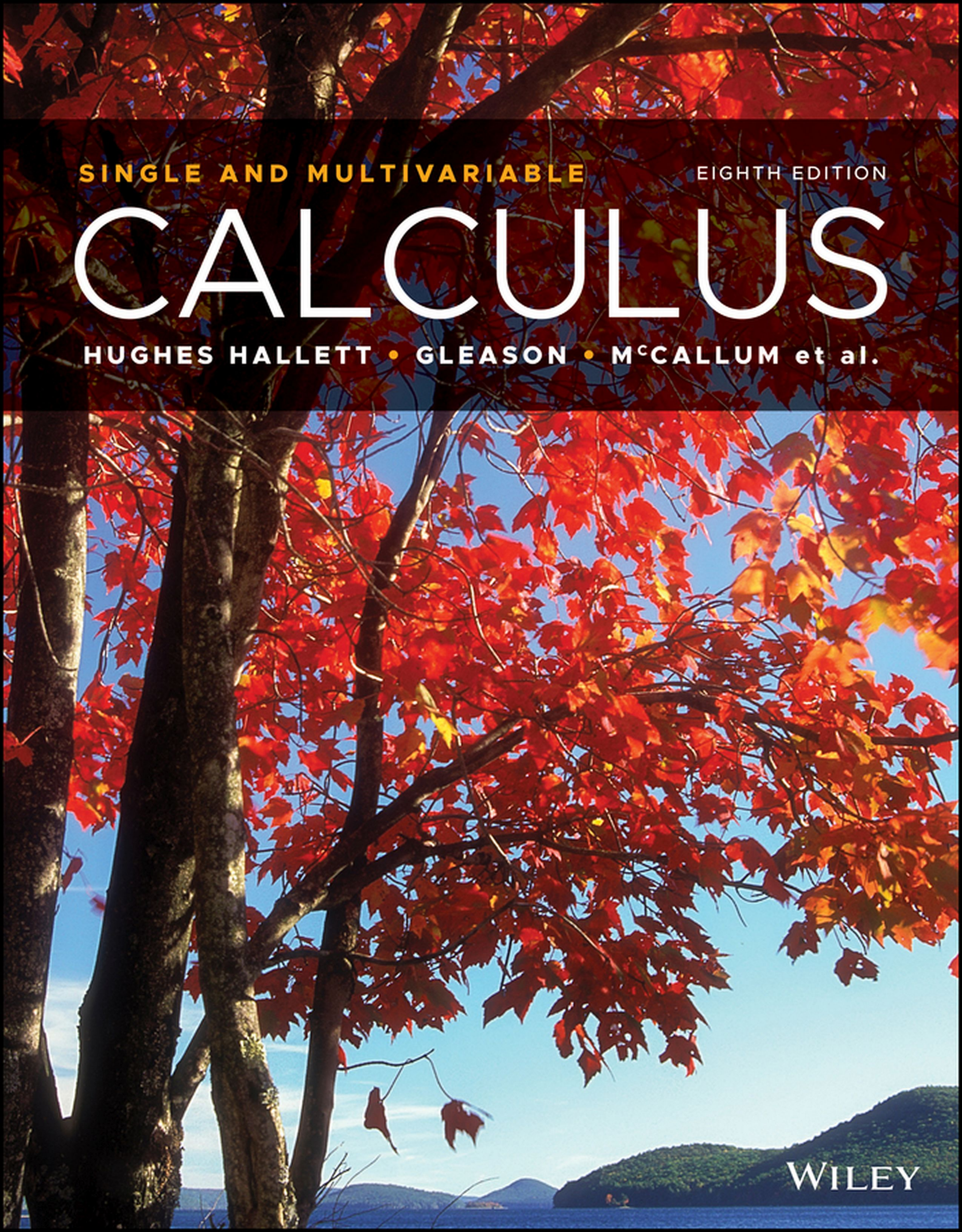

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Lines

Slope of line through (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through (x_1, y_1) with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = b + mx$$

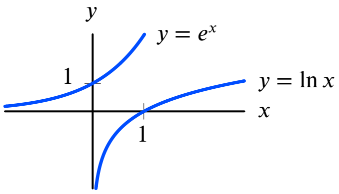
Rules of Exponents

$$a^x a^t = a^{x+t}$$
$$\frac{a^x}{a^t} = a^{x-t}$$
$$(a^x)^t = a^{xt}$$

Definition of Natural Log

$y = \ln x$ means $e^y = x$

ex: $\ln 1 = 0$ since $e^0 = 1$



Identities

$$\ln e^x = x$$
$$e^{\ln x} = x$$

Rules of Natural Logarithms

$$\ln(AB) = \ln A + \ln B$$
$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$
$$\ln A^p = p \ln A$$

Distance and Midpoint Formulas

Distance D between (x_1, y_1) and (x_2, y_2):

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of (x_1, y_1) and (x_2, y_2):

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring Special Polynomials

$x^2 - y^2 = (x + y)(x - y)$
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

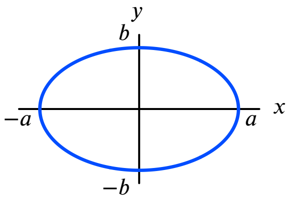
Circles

Center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

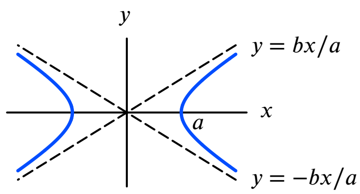
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Geometric Formulas

Conversion Between Radians and Degrees: π radians $= 180°$

Triangle

$A = \frac{1}{2}bh$

$= \frac{1}{2}ab\sin\theta$

Circle

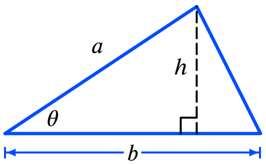
$A = \pi r^2$

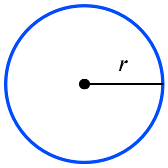
$C = 2\pi r$

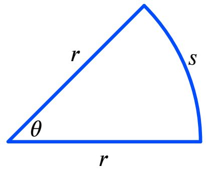
Sector of Circle

$A = \frac{1}{2}r^2\theta$ (θ in radians)

$s = r\theta$ (θ in radians)







Sphere

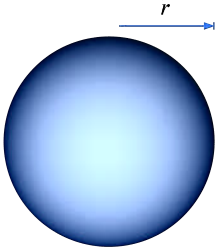
$V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$

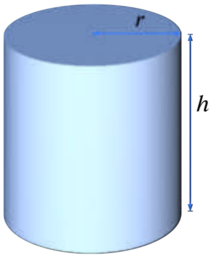
Cylinder

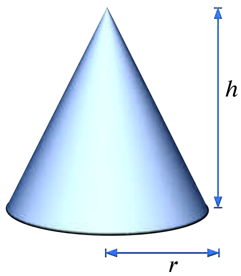
$V = \pi r^2 h$

Cone

$V = \frac{1}{3}\pi r^2 h$







Trigonometric Functions

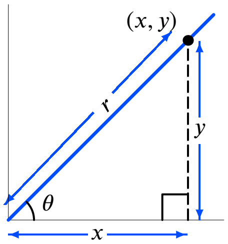
$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos^2\theta + \sin^2\theta = 1$

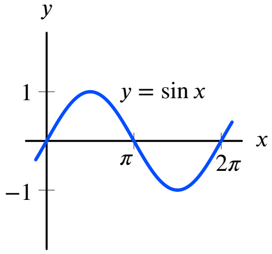


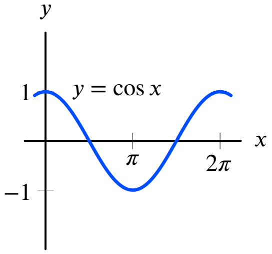
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

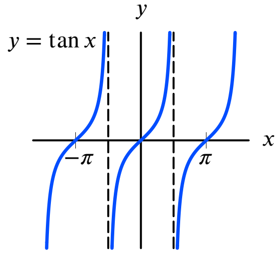
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\sin(2A) = 2\sin A\cos A$

$\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A$







The Binomial Theorem

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots + nxy^{n-1} + y^n$$

$$(x-y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots \pm nxy^{n-1} \mp y^n$$

CALCULUS

Eighth Edition

We dedicate this book to Andrew M. Gleason.

His brilliance and the extraordinary kindness and dignity with which he treated others made an enormous difference to us, and to many, many people. Andy brought out the best in everyone.

Deb Hughes Hallett
for the Calculus Consortium

CALCULUS

Eighth Edition

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PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful both because its central theme—change—is pivotal to an analysis of the natural world and because of its extraordinary power to reduce complicated problems to simple procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Calculus* continues our effort to promote courses in which understanding and computation reinforce each other. It reflects the input of users at research universities, four-year colleges, community colleges, and secondary schools, as well as of professionals in partner disciplines such as engineering and the natural and social sciences.

Flexibility in a New Era of Teaching and Learning

The world has changed and the education system has changed with it. With little or no training, instructors and students have adjusted to distance learning. As instructors ourselves, we saw how challenging this adjustment has been, especially for our students. These experiences taught us first-hand the importance of being able to adapt our classes to a variety of formats, from regular in-person classes, to online courses or some hybrid version in-between. The basis of the Eighth Edition is to provide a text and companion resources that are flexible enough to support an active and engaging experience in each of these formats.

Active Learning: Good Problems in Different Formats

Active participation in solving well-crafted problems promotes student learning. Since its inception, the hallmark of our text has been its innovative and engaging problems. These problems probe student understanding in ways often taken for granted. Praised for their creativity and variety, these problems have had influence far beyond the users of our textbook.

The Eighth Edition continues this tradition by providing an array of new problems, with many drawing on data from timely real-world applications. The Eighth Edition also expands on this tradition by adapting existing and new problems into an online format that retains the original pedagogical goals of the problem. Under our approach, which we call the "Rule of Four," ideas are presented graphically, numerically, symbolically, and verbally, thereby encouraging students to deepen their understanding.

Problems types in this text include:

- **Strengthen Your Understanding** problems at the end of every section. These problems ask students to reflect on what they have learned by deciding "What is wrong?" with a statement and to "Give an example" of an idea. Many of these problems have been adapted into WileyPLUS.
- **ConcepTests** promote active learning in the classroom. These can be used with polling software, and have been shown to dramatically improve student learning. Available at www.WileyPLUS.com. All ConcepTests have been adapted into WileyPLUS problems so can serve as a tool to measure student understanding in a virtual classroom.
- **Class Worksheets** allow instructors to engage students in individual or group class-work. Samples are available in the Instructor's Manual, and at www.WileyPLUS.com.
- **Data and Models** Many examples and problems throughout the text involve data-driven models. For example, Section 8.6 begins with applications of present and future value to new studies on the costs of climate change.
- **Drill Exercises** in the text, with most adapted into WileyPLUS, can be used to build student skill and confidence both in the classroom and virtually.

Mathematical Thinking Supported by Theory and Modeling

The first stage in the development of mathematical thinking is the acquisition of a clear intuitive picture of the central ideas. In the next stage, the student learns to reason with the intuitive ideas in plain English. After this foundation has been laid, there is a choice of direction. All students benefit from both theory and modeling, but the balance may differ for different groups. Some students, such as mathematics majors, may prefer more theory, while others may prefer more modeling. For instructors wishing to emphasize the connection between calculus and other fields, the text includes:

- A variety of problems from the **physical sciences** and **engineering**.
- Examples from the **biological sciences** and **economics**.
- Models from the **health sciences** and of **population growth**.
- Problems on **sustainability** and **climate change**.
- Case studies on **medicine** by David E. Sloane, MD.

Enhanced Online Content

This Eigth Edition provides opportunities for students to experience the concepts of calculus in ways that are not possible in a traditional textbook. The E-Text of *Calculus*, powered by VitalSource, and WileyPLUS provide a wealth of resources such as interactive demonstrations of concepts, embedded videos that illustrate problem-solving techniques, and built-in assessments that allow students to check their understanding as they read.

Specific resources include:

- Concise introductory **videos** for every section.
- Customizable animated **PowerPoint** slides giving a short introduction for every section.
- Worked example **videos** by Donna Krawczyk at the University of Arizona, which provide students the opportunity to see and hear hundreds of the book's examples being explained in detail.
- Homework management tools, which enable the instructor to assign questions easily and grade them automatically, using a rich set of options and controls.
- Pre-designed homework assignments. Use them as-is or customize them to fit the needs of your classroom.
- Set up for Success questions, in which students are prompted for responses as they step through a problem solution and receive targeted feedback based on those responses.
- Algebra & Trigonometry Refresher material provides students with an opportunity to brush up on material necessary to master Calculus.
- Embedded **Interactive Explorations**, applets that present and explore key ideas graphically and dynamically—especially useful for display of three-dimensional graphs.
- Material that reviews and extends the major ideas of each chapter: Extra problems for many section, Review Exercises and Problems for each chapter, CAS Challenge Problems, and Projects.
- Challenging problems that involve further exploration and application.
- Section on the ϵ, δ definition of limit (1.10).
- Appendices that include preliminary ideas useful in this course.

Flexibility and Adaptability: Varied Approaches

The Eighth Edition of *Calculus* is designed to provide flexibility for instructors who have a range of preferences regarding inclusion of topics and applications and the use of computational technology. For those who prefer the lean topic list of earlier editions, we have kept clear the main conceptual paths. For example,

- The Key Concept chapters on the derivative and the definite integral (Chapters 2 and 5) can be covered at the outset of the course, right after Chapter 1.
- Limits and continuity (Sections 1.7, 1.8, and 1.9) can be covered in depth before the introduction of the derivative (Sections 2.1 and 2.2), or after.
- Approximating Functions Using Series (Chapter 10) can be covered before, or without, Chapter 9.
- In Chapter 4 (Using the Derivative), instructors can select freely from Sections 4.3–4.8.
- Chapter 8 (Using the Definite Integral) contains a wide range of applications. Instructors can select one or two to do in detail.
- A Fundamental Tool: Vectors (Chapter 13) can be covered before Chapter 12 (Functions of Several Variables).
- Instructors can teach a course in Multivariable Calculus using Chapters 12–16, or a course in Vector Calculus using Chapters 12–14 and a selection of material from Chapters 17–21.
- Instructors who want to show how to calculate flux integrals using general parameterizations early can teach Chapter 21 (Parameters, Coordinates and Integrals) after Section 19.1.

To use calculus effectively, students need skill in both symbolic manipulation and the use of technology. The balance between the two may vary, depending on the needs of the students and the wishes of the instructor. The book is adaptable to many different combinations.

The book does not require any specific software or technology. It has been used with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice. Students are expected to use their own judgment to determine where technology is useful.

Content

This content represents our vision of how calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

Changes to the text in the Eighth Edition are in italics. In all chapters, problems were added and others updated. In total there are more than 300 new problems, with a particular emphasis on ones that are moderately difficult and computational.

Chapter 1: A Library of Functions

This chapter introduces all the elementary functions to be used in the book. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them may be new. We introduce exponential functions at the earliest possible stage, since they are fundamental to the understanding of real-world processes.

Section 1.7 now includes a wider variety of problems on calculating limits that can be solved using algebra or estimated using a graph.

Chapter 2: Key Concept: The Derivative

The purpose of this chapter is to give the student a practical understanding of the definition of the derivative and its interpretation as an instantaneous rate of change. The power rule is introduced; other rules are introduced in Chapter 3.

Chapter 3: Short-Cuts to Differentiation

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products; quotients; and composite, inverse, hyperbolic, and implicitly defined functions.

Chapter 4: Using the Derivative

The aim of this chapter is to enable the student to use the derivative in solving problems, including optimization, graphing, rates, parametric equations, and indeterminate forms. It is not necessary to cover all the sections in this chapter.

More optimization problems in Economics have been added to Section 4.3, including average cost and trade-off problems. To help students in better understanding related rates, more problems have been added in Section 4.6, where a rate needs to be calculated, but the equations and rates are given.

Chapter 5: Key Concept: The Definite Integral

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus.

Chapter 6: Constructing Antiderivatives

This chapter focuses on going backward from a derivative to the original function, first graphically and numerically, then analytically. It introduces the Second Fundamental Theorem of Calculus and the concept of a differential equation.

Chapter 7: Integration

This chapter includes several techniques of integration, including substitution, parts, partial fractions, and trigonometric substitutions; others are included in the table of integrals. There are discussions of numerical methods and of improper integrals.

Problems have been added in Sections 7.1, 7.2, and 7.6 to emphasis the form of different integrands.

Chapter 8: Using the Definite Integral

This chapter emphasizes the idea of subdividing a quantity to produce Riemann sums which, in the limit, yield a definite integral. It shows how the integral is used in geometry, physics, economics, and probability; polar coordinates are introduced. It is not necessary to cover all the sections in this chapter.

Section 8.6 has been rewritten to introduce the ideas of present and future value using the predicted costs of climate change, with related problems.

Chapter 9: Sequences and Series

This chapter focuses on sequences, series of constants, and convergence. It includes the integral, ratio, comparison, limit comparison, and alternating series tests. It also introduces geometric series and general power series, including their intervals of convergence.

Chapter 10: Approximating Functions

This chapter introduces Taylor Series and Fourier Series using the idea of approximating functions by simpler functions.

Chapter 11: Differential Equations

This chapter introduces differential equations. The emphasis is on qualitative solutions, modeling, and interpretation.

Chapter 12: Functions of Several Variables

This chapter introduces functions of many variables from several points of view, using surface graphs, contour diagrams, and tables. We assume throughout that functions of two or more variables are defined on regions with piecewise smooth boundaries. We conclude with a section on continuity. Chapter 13 can be taught before Chapter 12.

Chapter 13: A Fundamental Tool: Vectors

This chapter introduces vectors geometrically and algebraically and discusses the dot and cross product. Chapter 13 can be taught before Chapter 12.

Chapter 14: Differentiating Functions of Several Variables

Partial derivatives, directional derivatives, gradients, and local linearity are introduced. The chapter also discusses higher order partial derivatives, quadratic Taylor approximations, and differentiability.

Chapter 15: Optimization

The ideas of the previous chapter are applied to optimization problems, both constrained and unconstrained.

Chapter 16: Integrating Functions of Several Variables

This chapter discusses double and triple integrals in Cartesian, polar, cylindrical, and spherical coordinates.

Chapter 17: Parameterization and Vector Fields

This chapter discusses parameterized curves and motion, vector fields and flowlines.

Chapter 18: Line Integrals

This chapter introduces line integrals and shows how to calculate them using parameterizations. Conservative fields, gradient fields, the Fundamental Theorem of Calculus for Line Integrals, and Green's Theorem are discussed.

Chapter 19: Flux Integrals and Divergence

This chapter introduces flux integrals and shows how to calculate them over surface graphs, portions of cylinders, and portions of spheres. The divergence is introduced and its relationship to flux integrals discussed in the Divergence Theorem.

Chapter 20: The Curl and Stokes' Theorem

The purpose of this chapter is to give students a practical understanding of the curl and of Stokes' Theorem and to lay out the relationship between the theorems of vector calculus.

Chapter 21: Parameters, Coordinates, and Integrals

This chapter covers parameterized surfaces, the change of variable formula in a double or triple integral, and flux though a parameterized surface.

Appendices

There are online appendices on roots, accuracy, and bounds; complex numbers; Newton's method; and vectors in the plane. The appendix on vectors can be covered at any time, but may be particularly useful in the conjunction with Section 4.8 on parametric equations.

Supplementary Materials and Additional Resources

Supplements for the instructor can be obtained online through WileyPLUS or by contacting your Wiley representative. The following supplementary materials are available for this edition:

- **Instructor's Manual** containing teaching tips, calculator programs, overhead transparency masters, sample worksheets, and sample syllabi.
- **Computerized Test Bank**, powered by TestGen, comprised of nearly 7,000 questions, mostly algorithmically-generated, which allows for multiple versions of a single test or quiz.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Student Solution Manual** with complete solutions to half the odd-numbered problems.
- **Graphing Calculator Manual**, to help students get the most out of their graphing calculators, and to show how they can apply the numerical and graphing functions of their calculators to their study of calculus.
- **Additional Material**, elaborating specially marked points in the text and password-protected electronic versions of the instructor ancillaries, can be found at www.WileyPLUS.com.

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Finally, we extend our particular thanks to Jon Christensen for his creativity with our three-dimensional figures.

The Calculus Consortium

To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. This book emphasizes at every stage the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Your success in using this book will depend on your reading, questioning, and thinking hard about the ideas presented. Although you may not have done this with other books, you should plan on reading the text in detail, not just the worked examples.
- There are very few examples in the text that are exactly like the homework problems. This means that you can't just look at a homework problem and search for a similar–looking "worked out" example. Success with the homework will come by grappling with the ideas of calculus.
- Many of the problems that we have included in the book are open-ended. This means that there may be more than one approach and more than one solution, depending on your analysis. Many times, solving a problem relies on common sense ideas that are not stated in the problem but which you will know from everyday life.
- Some problems in this book assume that you have access to a graphing calculator or computer. There are many situations where you may not be able to find an exact solution to a problem, but you can use a calculator or computer to get a reasonable approximation.
- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values) algebraic (a formula), and verbal. Sometimes you may find it easier to translate a problem given in one form into another. The best idea is to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups very helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the most important accomplishments of the millennium—calculus—as well as a real sense of the power of mathematics in the age of technology.

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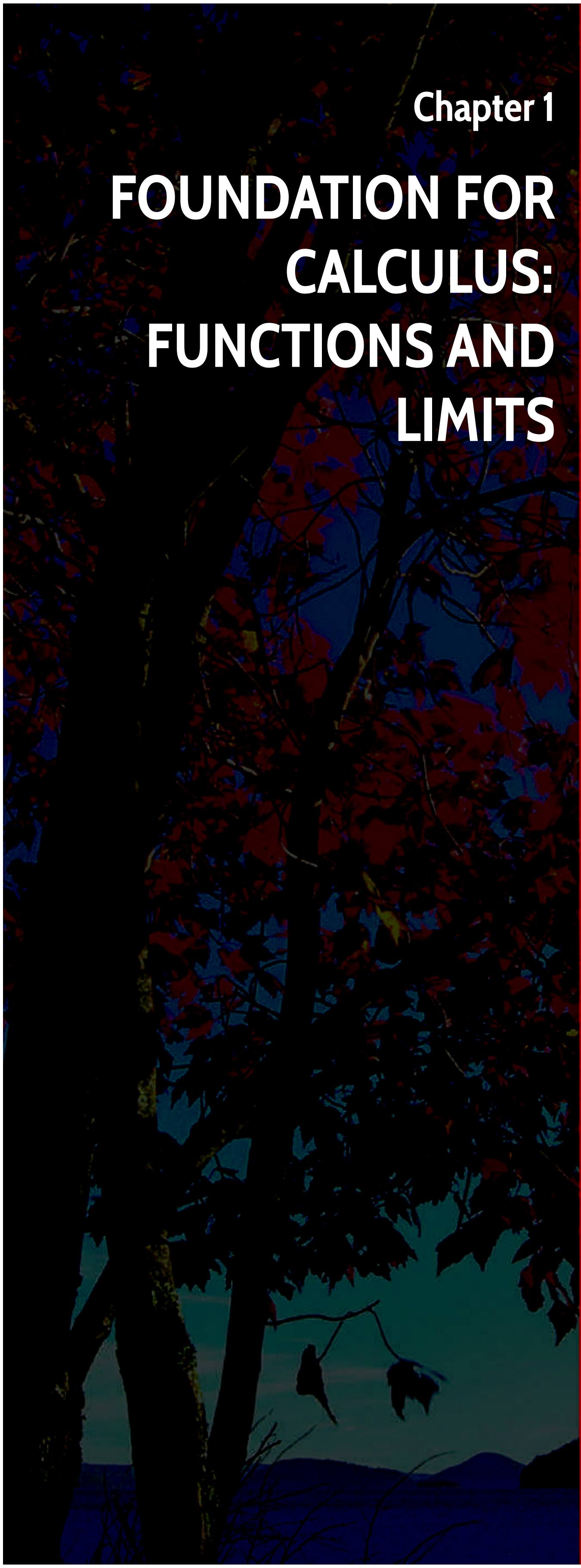


Chapter 1

FOUNDATION FOR CALCULUS: FUNCTIONS AND LIMITS

CONTENTS

1.1 FUNCTIONS AND CHANGE

In mathematics, a *function* is used to represent the dependence of one quantity upon another.

Let's look at an example. In 2015, Boston, Massachusetts, had the highest annual snowfall, 110.6 inches, since recording started in 1872. Table 1.1 shows one 14-day period in which the city broke another record with a total of 64.4 inches.[1]

Table 1.1 *Daily snowfall in inches for Boston, January 27 to February 9, 2015*

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Snowfall	22.1	0.2	0	0.7	1.3	0	16.2	0	0	0.8	0	0.9	7.4	14.8

You may not have thought of something so unpredictable as daily snowfall as being a function, but it *is* a function of day, because each day gives rise to one snowfall total. There is no formula for the daily snowfall (otherwise we would not need a weather bureau), but nevertheless the daily snowfall in Boston does satisfy the definition of a function: Each day, t, has a unique snowfall, S, associated with it.

We define a function as follows:

> A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

The input is called the *independent variable* and the output is called the *dependent variable*. In the snowfall example, the domain is the set of days $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ and the range is the set of daily snowfalls $\{0, 0.2, 0.7, 0.8, 0.9, 1.3, 7.4, 14.8, 16.2, 22.1\}$. We call the function f and write $S = f(t)$. Notice that a function may have identical outputs for different inputs (Days 8 and 9, for example).

Some quantities, such as a day or date, are *discrete*, meaning they take only certain isolated values (days must be integers). Other quantities, such as time, are *continuous* as they can be any number. For a continuous variable, domains and ranges are often written using interval notation:

The set of numbers t such that $a \leq t \leq b$ is called a *closed interval* and written $[a, b]$.

The set of numbers t such that $a < t < b$ is called an *open interval* and written (a, b).

The Rule of Four: Tables, Graphs, Formulas, and Words

Functions can be represented by tables, graphs, formulas, and descriptions in words. For example, the function giving the daily snowfall in Boston can be represented by the graph in Figure 1.1, as well as by Table 1.1.

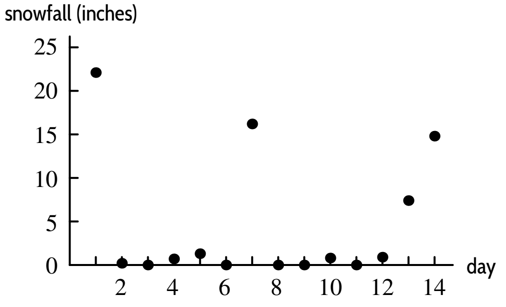


Figure 1.1: Boston snowfall, starting January 27, 2015

As another example of a function, consider the snowy tree cricket. Surprisingly enough, all such crickets chirp at essentially the same rate if they are at the same temperature. That means that the chirp rate is a function of temperature. In other words, if we know the temperature, we can determine

[1] w2.weather.gov/climate/xmacis.php?wfo=box, accessed June 2015.

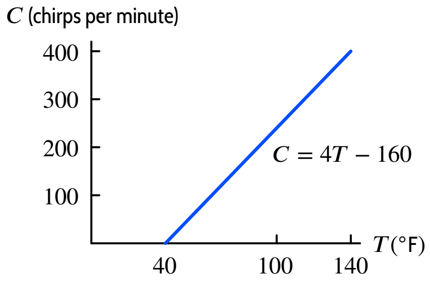


Figure 1.2: Cricket chirp rate versus temperature

the chirp rate. Even more surprisingly, the chirp rate, C, in chirps per minute, increases steadily with the temperature, T, in degrees Fahrenheit, and can be computed by the formula

$$C = 4T - 160$$

to a fair level of accuracy. We write $C = f(T)$ to express the fact that we think of C as a function of T and that we have named this function f. The graph of this function is in Figure 1.2.

Notice that the graph of $C = f(T)$ in Figure 1.2 is a solid line. This is because $C = f(T)$ is a *continuous function*. Roughly speaking, a continuous function is one whose graph has no breaks, jumps, or holes. This means that the independent variable must be continuous. (We give a more precise definition of continuity of a function in Section 1.7.)

Examples of Domain and Range

If the domain of a function is not specified, we usually take it to be the largest possible set of real numbers. For example, we usually think of the domain of the function $f(x) = x^2$ as all real numbers. However, the domain of the function $g(x) = 1/x$ is all real numbers except zero, since we cannot divide by zero.

Sometimes we restrict the domain to be smaller than the largest possible set of real numbers. For example, if the function $f(x) = x^2$ is used to represent the area of a square of side x, we restrict the domain to nonnegative values of x.

Example 1 The function $C = f(T)$ gives chirp rate as a function of temperature. We restrict this function to temperatures for which the predicted chirp rate is positive, and up to the highest temperature ever recorded at a weather station, 134°F.[2] What is the domain of this function f?

Solution If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables C and T, any T value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then C cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40$°F, we see that T cannot be less than 40°F. (See Figure 1.2.) In addition, we are told that the function is not defined for temperatures above 134°. Thus, for the function $C = f(T)$ we have

$$\begin{aligned} \text{Domain} &= \text{All } T \text{ values between } 40°\text{F and } 134°\text{F} \\ &= \text{All } T \text{ values with } 40 \le T \le 134 \\ &= [40, 134]. \end{aligned}$$

Example 2 Find the range of the function f, given the domain from Example 1. In other words, find all possible values of the chirp rate, C, in the equation $C = f(T)$.

Solution Again, if we consider $C = 4T - 160$ simply as a mathematical relationship, its range is all real C values. However, when thinking of the meaning of $C = f(T)$ for crickets, we see that the function predicts cricket chirps per minute between 0 (at $T = 40$°F) and 376 (at $T = 134$°F). Hence,

$$\begin{aligned} \text{Range} &= \text{All } C \text{ values from 0 to 376} \\ &= \text{All } C \text{ values with } 0 \le C \le 376 \\ &= [0, 376]. \end{aligned}$$

[2] www.guinnessworldrecords.com, accessed January 2017.

In using the temperature to predict the chirp rate, we thought of the temperature as the *independent variable* and the chirp rate as the *dependent variable*. However, we could do this backward, and calculate the temperature from the chirp rate. From this point of view, the temperature is dependent on the chirp rate. Thus, which variable is dependent and which is independent may depend on your viewpoint.

Linear Functions

The chirp-rate function, $C = f(T)$, is an example of a *linear function*. A function is linear if its slope, or rate of change, is the same at every point. The rate of change of a function that is not linear may vary from point to point.

Olympic and World Records

During the early years of the Olympics, the height of the men's winning pole vault increased approximately 20 cm every four years. Table 1.2 shows that the height started at 330 cm in 1900, and increased by the equivalent of 5 cm a year. So the height was a linear function of time from 1900 to 1912. If y is the winning height in centimeters and t is the number of years since 1900, then y is predicted approximately by

$$y = f(t) = 330 + 5t.$$

Since $y = f(t)$ increases with t, we say that f is an *increasing function.* The coefficient 5 tells us the rate, in centimeters per year, at which the height increases.

Table 1.2 *Men's Olympic pole vault winning height (approximate)*

Year	1900	1904	1908	1912
Height (centimeters)	330	350	370	390

This rate of increase is the *slope* of the line in Figure 1.3. The slope is given by the ratio

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{370 - 350}{8 - 4} = \frac{20}{4} = 5 \text{ centimeters/year.}$$

Calculating the slope (rise/run) using any other two points on the line gives the same value.

What about the constant 330? This represents the initial height in 1900, when $t = 0$. Geometrically, 330 is the *intercept* on the vertical axis.

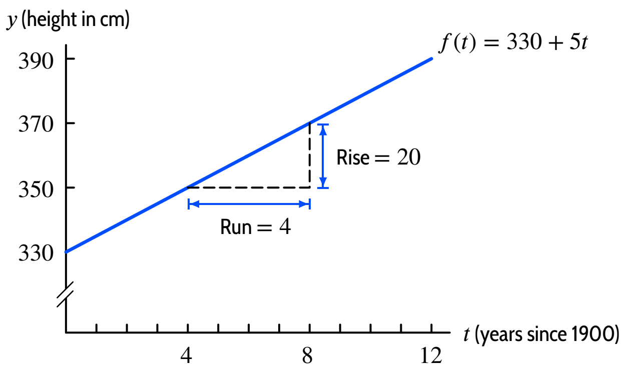


Figure 1.3: Olympic pole vault records

You may wonder whether the linear trend continues beyond 1912. Not surprisingly, it does not give a good prediction. The formula $y = 330 + 5t$ predicts that the height in the 2016 Olympics would be 910 centimeters, which is considerably higher than the actual value of 603 centimeters. There is clearly a danger in *extrapolating* too far from the given data. You should also observe that the data in Table 1.2 is discrete, because it is given only at specific points (every four years). However, we have treated the variable t as though it were continuous, because the function $y = 330 + 5t$ makes sense for all values of t. The graph in Figure 1.3 is of the continuous function because it is a solid line, rather than four separate points representing the years in which the Olympics were held.

As the pole vault heights have increased over the years, the time to run the mile has decreased. If y is the world record time to run the mile, in seconds, and t is the number of years since 1900, then records show that, approximately,

$$y = g(t) = 260 - 0.39t.$$

The 260 tells us that the world record was 260 seconds in 1900 (at $t = 0$). The slope, -0.39, tells us that the world record decreased by about 0.39 seconds per year. We say that g is a *decreasing function*.

Difference Quotients and Delta Notation

We use the symbol Δ (the Greek letter capital delta) to mean "change in," so Δx means change in x and Δy means change in y.

The slope of a linear function $y = f(x)$ can be calculated from values of the function at two points, given by x_1 and x_2, using the formula

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

The quantity $(f(x_2) - f(x_1))/(x_2 - x_1)$ is called a *difference quotient* because it is the quotient of two differences. (See Figure 1.4.) Since $m = \Delta y/\Delta x$, the units of m are y-units over x-units.

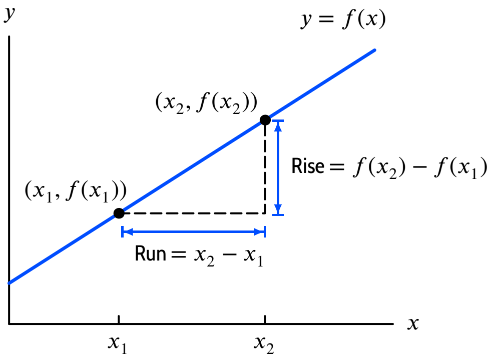


Figure 1.4: Difference quotient $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

Families of Linear Functions

A **linear function** has the form

$$y = f(x) = b + mx.$$

Its graph is a line such that
- m is the **slope**, or rate of change of y with respect to x.
- b is the **vertical intercept**, or value of y when x is zero.

Notice that if the slope, m, is zero, we have $y = b$, a horizontal line.

To recognize that a table of x and y values comes from a linear function, $y = b + mx$, look for differences in y-values that are constant for equally spaced x-values.

Formulas such as $f(x) = b + mx$, in which the constants m and b can take on various values, give a *family of functions*. All the functions in a family share certain properties—in this case, all the graphs are straight lines. The constants m and b are called *parameters*; their meaning is shown in Figures 1.5 and 1.6. Notice that the greater the magnitude of m, the steeper the line.

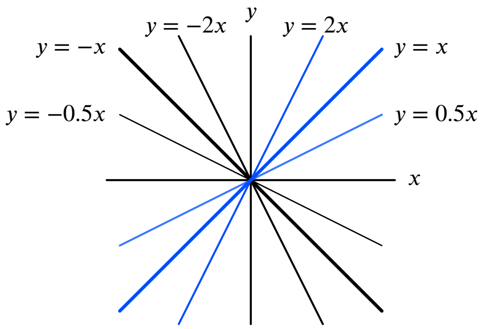


Figure 1.5: The family $y = mx$ (with $b = 0$)

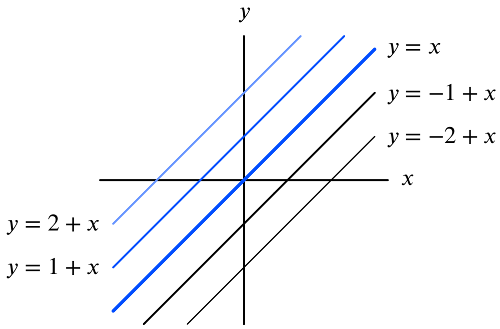


Figure 1.6: The family $y = b + x$ (with $m = 1$)

Increasing versus Decreasing Functions

The terms increasing and decreasing can be applied to other functions, not just linear ones. See Figure 1.7. In general,

> A function f is **increasing** if the values of $f(x)$ increase as x increases.
> A function f is **decreasing** if the values of $f(x)$ decrease as x increases.
>
> The graph of an *increasing* function *climbs* as we move from left to right.
> The graph of a *decreasing* function *falls* as we move from left to right.
>
> A function $f(x)$ is **monotonic** if it increases for all x or decreases for all x.

Figure 1.7: Increasing and decreasing functions

Proportionality

A common functional relationship occurs when one quantity is *proportional* to another. For example, the area, A, of a circle is proportional to the square of the radius, r, because

$$A = f(r) = \pi r^2.$$

> We say y is (directly) **proportional** to x if there is a nonzero constant k such that
>
> $$y = kx.$$
>
> This k is called the constant of proportionality.

We also say that one quantity is *inversely proportional* to another if one is proportional to the reciprocal of the other. For example, the speed, v, at which you make a 50-mile trip is inversely proportional to the time, t, taken, because v is proportional to $1/t$:

$$v = 50\left(\frac{1}{t}\right) = \frac{50}{t}.$$

Summary for Section 1.1

- **Definition of function**: a rule which takes numbers as inputs and assigns to each input exactly one output number.
- The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.
- **Function notation:** $y = f(t)$, where t is the **independent variable** and y is the **dependent variable**.
- A **linear function** has values of y that change at a constant rate with the values of x.
- **Formula** for linear functions:

$$y = \underbrace{b}_{\text{Initial value}} + \underbrace{m}_{\text{Slope}} \cdot x$$

- The graph of a linear function is a **line**.
  - b is the **vertical intercept**, or ***y*-intercept**, and gives the value of y for $x = 0$.
  - m is the **slope** of the line, and gives the rate of change of y with respect to x:

$$m = \frac{\Delta y}{\Delta x}.$$

- Formulas such as $f(x) = b + mx$, in which the constants m and b can take on various values, represent a **family of functions**.
- A function f is **increasing** if the values of $f(x)$ increase as x increases.
- A function f is **decreasing** if the values of $f(x)$ decrease as x increases.
- A function f is **monotonic** if it increases for all x or decreases for all x.
- **Proportionality:** We say y is (directly) **proportional** to x if there is a nonzero constant k such that $y = kx$ and k is called the constant of proportionality.

Exercises and Problems for Section 1.1

EXERCISES

1. The population of a city, P, in millions, is a function of t, the number of years since 2020, so $P = f(t)$. Explain the meaning of the statement $f(5) = 8$ in terms of the population of this city.

2. The pollutant PCB (polychlorinated biphenyl) can affect the thickness of pelican eggshells. Thinking of the thickness, T, of the eggshells, in mm, as a function of the concentration, P, of PCBs in ppm (parts per million), we have $T = f(P)$. Explain the meaning of $f(200)$ in terms of thickness of pelican eggs and concentration of PCBs.

3. Describe what Figure 1.8 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

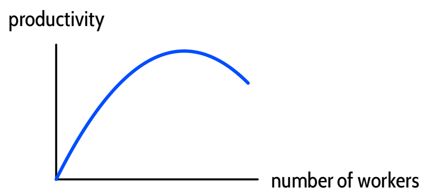


Figure 1.8

4. Match the graphs in Figure 1.9 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = x - 5$ **(b)** $-3x + 4 = y$
(c) $5 = y$ **(d)** $y = -4x - 5$
(e) $y = x + 6$ **(f)** $y = x/2$

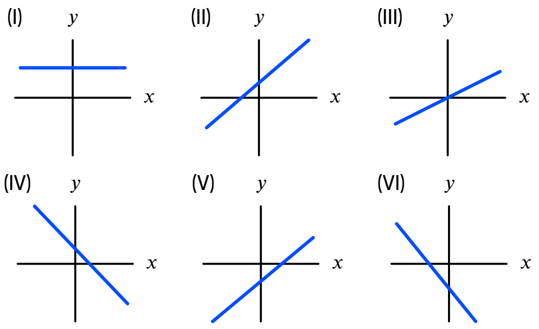


Figure 1.9

5. Match the graphs in Figure 1.10 with the following equations. (Note that the x and y scales may be unequal.)

(a) $y = -2.72x$ **(b)** $y = 0.01 + 0.001x$
(c) $y = 27.9 - 0.1x$ **(d)** $y = 0.1x - 27.9$
(e) $y = -5.7 - 200x$ **(f)** $y = x/3.14$

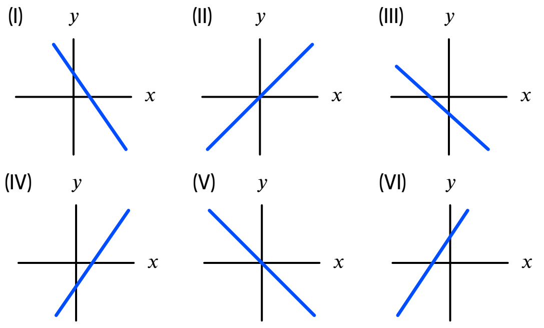


Figure 1.10

■ For Exercises 6–9, find an equation for the line that passes through the given points.

6. $(0, 0)$ and $(1, 1)$ **7.** $(0, 2)$ and $(2, 3)$
8. $(-2, 1)$ and $(2, 3)$ **9.** $(-1, 0)$ and $(2, 6)$

■ For Exercises 10–13, determine the slope and the y-intercept of the line whose equation is given.

10. $2y + 5x - 8 = 0$ **11.** $7y + 12x - 2 = 0$
12. $-4y + 2x + 8 = 0$ **13.** $12x = 6y + 4$

14. Estimate the slope and the equation of the line in Figure 1.11.

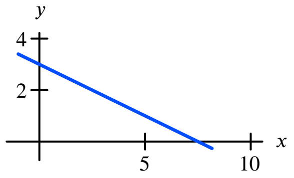


Figure 1.11

15. Find an equation for the line with slope m through the point (a, c).

16. Find a linear function that generates the values in Table 1.3.

Table 1.3

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

■ For Exercises 17–19, use the facts that parallel lines have equal slopes and that the slopes of perpendicular lines are negative reciprocals of one another.

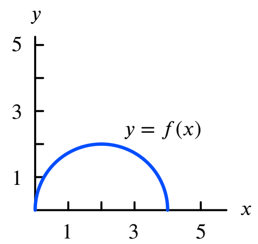
17. Find an equation for the line through the point $(2, 1)$ which is perpendicular to the line $y = 5x - 3$.

18. Find equations for the lines through the point $(1, 5)$ that are parallel to and perpendicular to the line with equation $y + 4x = 7$.

19. Find equations for the lines through the point (a, b) that are parallel and perpendicular to the line $y = mx + c$, assuming $m \neq 0$.

■ For Exercises 20–23, give the approximate domain and range of each function. Assume the entire graph is shown.

20.



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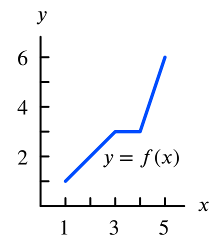

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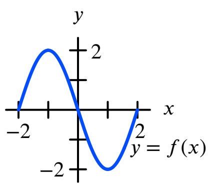

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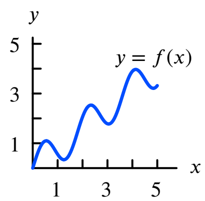

■ Find the domain and range in Exercises 24–25.

24. $y = x^2 + 2$ **25.** $y = \dfrac{1}{x^2 + 2}$

26. If $f(t) = \sqrt{t^2 - 16}$, find all values of t for which $f(t)$ is a real number. Solve $f(t) = 3$.

■ In Exercises 27–31, write a formula representing the function.

27. The volume of a sphere is proportional to the cube of its radius, r.

28. The average velocity, v, for a trip over a fixed distance, d, is inversely proportional to the time of travel, t.

29. The strength, S, of a beam is proportional to the square of its thickness, h.

30. The energy, E, expended by a swimming dolphin is proportional to the cube of the speed, v, of the dolphin.

31. The number of animal species, N, of a certain body length, l, is inversely proportional to the square of l.

PROBLEMS

32. In December 2010, the snowfall in Minneapolis was unusually high,[3] leading to the collapse of the roof of the Metrodome. Figure 1.12 gives the snowfall, S, in Minneapolis for December 6–15, 2010.

(a) How do you know that the snowfall data represents a function of date?
(b) Estimate the snowfall on December 12.
(c) On which day was the snowfall more than 10 inches?
(d) During which consecutive two-day interval was the increase in snowfall largest?

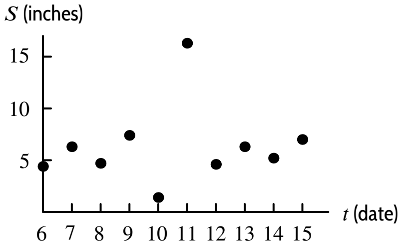

Figure 1.12

33. The value of a car, $V = f(a)$, in thousands of dollars, is a function of the age of the car, a, in years.

(a) Interpret the statement $f(5) = 6$.
(b) Sketch a possible graph of V against a. Is f an increasing or decreasing function? Explain.
(c) Explain the significance of the horizontal and vertical intercepts in terms of the value of the car.

34. Which graph in Figure 1.13 best matches each of the following stories?[4] Write a story for the remaining graph.

(a) I had just left home when I realized I had forgotten my books, so I went back to pick them up.
(b) Things went fine until I had a flat tire.
(c) I started out calmly but sped up when I realized I was going to be late.

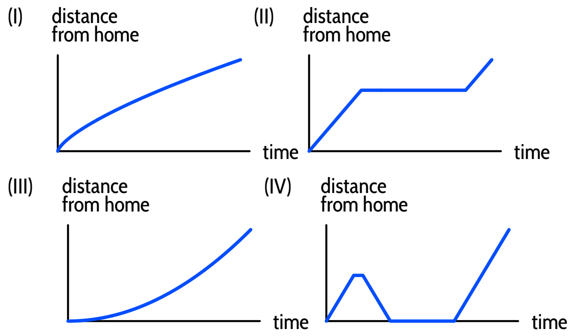

Figure 1.13

■ In Problems 35–38, the function $S = f(t)$ gives the average annual sea level, S, in meters above a fixed reference level, in Aberdeen, Scotland,[5] as a function of t, the number of years before 2020. Write a mathematical expression that represents the given statement.

35. In 2018 the average annual sea level in Aberdeen was 7.088 meters.

36. The average annual sea level in Aberdeen in 2020.

37. The average annual sea level in Aberdeen was the same in 1949 and 2000.

38. The average annual sea level in Aberdeen decreased by 11 millimeters from 2017 to 2018.

■ Problems 39–42 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."

39. Driving speed against time on the highway

40. Distance driven against time on the highway

41. Distance from my exit vs time on the highway

42. Distance between cars vs distance driven on the highway

43. An object is put outside on a cold day at time $t = 0$ minutes. Its temperature, $H = f(t)$, in °C, is graphed in Figure 1.14.

(a) What does the statement $f(30) = 10$ mean in terms of temperature? Include units for 30 and for 10 in your answer.
(b) Explain what the vertical intercept, a, and the horizontal intercept, b, represent in terms of temperature of the object and time outside.

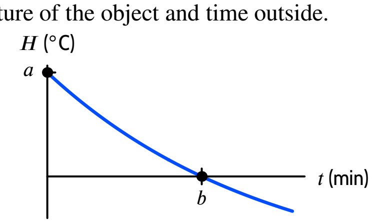

Figure 1.14

44. A rock is dropped from a window and falls to the ground below. The height, s (in meters), of the rock above ground is a function of the time, t (in seconds), since the rock was dropped, so $s = f(t)$.

(a) Sketch a possible graph of s as a function of t.
(b) Explain what the statement $f(7) = 12$ tells us about the rock's fall.
(c) The graph drawn as the answer for part (a) should have a horizontal and vertical intercept. Interpret each intercept in terms of the rock's fall.

[3] http://www.crh.noaa.gov/mpx/Climate/DisplayRecords.php
[4] Adapted from Jan Terwel, "Real Math in Cooperative Groups in Secondary Education", *Cooperative Learning in Mathematics*, ed. Neal Davidson, p. 234 (Reading: Addison Wesley, 1990).
[5] www.psmsl.org, accessed August 12, 2019.

45. You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.

46. US imports of crude oil and petroleum were increasing between 1992 and 2007.[6] There were many ups and downs, but the general trend is shown by the line in Figure 1.15.

(a) Find the slope of the line. Include its units of measurement.
(b) Write an equation for the line. Define your variables, including their units.
(c) Assuming the trend continues, when does the linear model predict imports will reach 18 million barrels per day? Do you think this is a reliable prediction? Give reasons.

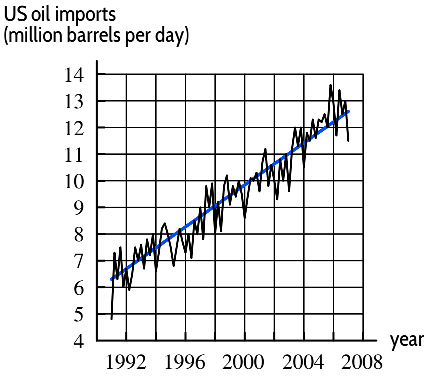

Figure 1.15

■ Problems 47–49 use Figure 1.16 showing how the quantity, Q, of grass (kg/hectare) in different parts of Namibia depended on the average annual rainfall, r, (mm), in two different years.[7]

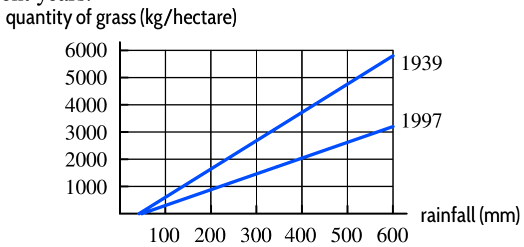

Figure 1.16

47. **(a)** For 1939, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

48. **(a)** For 1997, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

49. Which of the two functions in Figure 1.16 has the larger difference quotient $\Delta Q/\Delta r$? What does this tell us about grass in Namibia?

50. For t in years since 1950, the quantity of carbon dioxide in the atmosphere, in ppm (parts per million), is predicted to be

$$g(t) = 300 + 1.4t.$$

(a) Find $g(10)$, $g(20)$ and $g(100)$.
(b) Suppose another model predicts the quantity of carbon dioxide to be

$$h(t) = 300 + 1.5t.$$

Find $h(10)$, $h(20)$ and $h(100)$.
(c) Do the functions $g(t)$ and $h(t)$ differ in their intercept or in their slope? Does this lead to a larger difference in values of $g(t)$ and $h(t)$ for small t or for large t? (Use your answers to parts (a) and (b) to decide.)

51. Marmots are large squirrels that hibernate in the winter and come out in the spring. Figure 1.17 shows the date (days after Jan 1) that they are first sighted each year in Colorado as a function of the average minimum daily temperature for that year.[8]

(a) Find the slope of the line, including units.
(b) What does the sign of the slope tell you about marmots?
(c) Use the slope to determine how much difference 6°C warming makes to the date of first appearance of a marmot.
(d) Find the equation of the line.

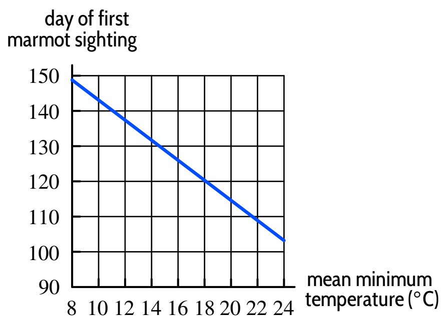

Figure 1.17

52. In Colorado spring has arrived when the bluebell first flowers. Figure 1.18 shows the date (days after Jan 1) that the first flower is sighted in one location as a function of the first date (days after Jan 1) of bare (snow-free) ground.[9]

(a) If the first date of bare ground is 140, how many days later is the first bluebell flower sighted?
(b) Find the slope of the line, including units.
(c) What does the sign of the slope tell you about bluebells?
(d) Find the equation of the line.

[6] www.theoildrum.com/node/2767, accessed August 16th, 2019.
[7] David Ward and Ben T. Ngairorue, "Are Namibia's Grasslands Desertifying?", *Journal of Range Management* 53, 2000, 138–144.
[8] David W. Inouye, Billy Barr, Kenneth B. Armitage, and Brian D. Inouye, "Climate Change is Affecting Altitudinal Migrants and Hibernating Species", *PNAS* 97, 2000, 1630–1633.
[9] *Ibid.*

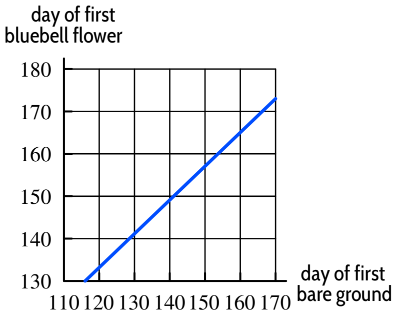

Figure 1.18

53. On March 5, 2015, Capracotta, Italy, received 256 cm (100.787 inches) of snow in 18 hours.[10]

(a) Assuming the snow fell at a constant rate and there were already 100 cm of snow on the ground, find a formula for $f(t)$, in cm, for the depth of snow as a function of t hours since the snowfall began on March 5.

(b) What are the domain and range of f?

54. In a Washington town, the charge for commercial waste collection is \$694.55 for 5 tons and \$1098.32 for 8 tons of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the weight, w, in tons.

(b) What is the slope of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.

(c) What is the vertical intercept of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.

55. For tax purposes, you may have to report the value of your assets, such as cars or refrigerators. The value you report drops with time. "Straight-line depreciation" assumes that the value is a linear function of time. If a \$950 refrigerator depreciates completely in seven years, find a formula for its value as a function of time.

56. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount monthly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed \$40, while one using 1600 cubic feet was billed \$55.

(a) What is the charge per cubic foot?

(b) Write an equation for the total cost of a resident's water as a function of cubic feet of water used.

(c) How many cubic feet of water used would lead to a bill of \$100?

57. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars are \$50 a day and 10 cents a mile.

(a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.

(b) On the same axes, graph both functions.

(c) How should you decide which company is cheaper?

58. A controversial 1992 Danish study[11] reported that men's average sperm count decreased from 113 million per milliliter in 1940 to 66 million per milliliter in 1990.

(a) Express the average sperm count, S, as a linear function of the number of years, t, since 1940.

(b) A man's fertility is affected if his sperm count drops below about 20 million per milliliter. If the linear model found in part (a) is accurate, in what year will the average male sperm count fall below this level?

59. Let $f(t)$ be the number of US billionaires in year t.

(a) Express the following statements[12] in terms of f.

(i) In 2001 there were 272 US billionaires.

(ii) In 2019 there were 607 US billionaires.

(b) Find the average yearly increase in the number of US billionaires from 2001 to 2019. Express this using f.

(c) Assuming the yearly increase remains constant, find a formula predicting the number of US billionaires in year t.

60. The cost of planting seed is usually a function of the number of acres sown. The cost of the equipment is a *fixed cost* because it must be paid regardless of the number of acres planted. The costs of supplies and labor vary with the number of acres planted and are called *variable costs*. Suppose the fixed costs are \$10,000 and the variable costs are \$200 per acre. Let C be the total cost, measured in thousands of dollars, and let x be the number of acres planted.

(a) Find a formula for C as a function of x.

(b) Graph C against x.

(c) Which feature of the graph represents the fixed costs? Which represents the variable costs?

61. An airplane uses a fixed amount of fuel for takeoff, a (different) fixed amount for landing, and a third fixed amount per mile when it is in the air. How does the total quantity of fuel required depend on the length of the trip? Write a formula for the function involved. Explain the meaning of the constants in your formula.

[10] iceagenow.info, accessed April 2015.
[11] "Investigating the Next Silent Spring," *US News and World Report*, pp. 50–52 (March 11, 1996).
[12] www.forbes.com/billionaires, accessed August 15, 2019.

62. For the line $y = f(x)$ in Figure 1.19, evaluate

(a) $f(423) - f(422)$ **(b)** $f(517) - f(513)$

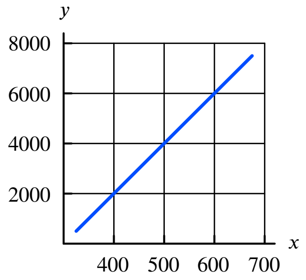

Figure 1.19

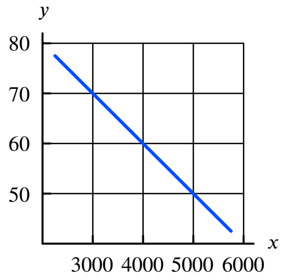

Figure 1.20

63. For the line $y = g(x)$ in Figure 1.20, evaluate

(a) $g(4210) - g(4209)$ **(b)** $g(3760) - g(3740)$

64. Let $V(t)$ be the average speed, in miles per hour, for a journey of 100 miles taking t hours. See Figure 1.21.

(a) Find $V(1)$ and $V(2)$. Give units.
(b) Do you think it is true for all t that

$$V(t+1) = V(t) + 1?$$

If so, give an argument. If not, give a counterexample.
(c) Answer the same question as part (b), but with

$$V(t+1) = V(t) + V(1).$$

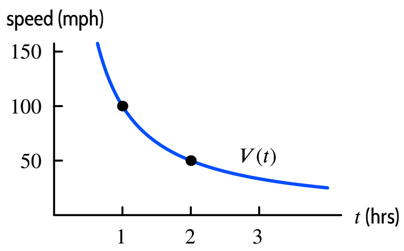

Figure 1.21

65. The percentage of people who have heard some news t days after it was announced is $N(t)$. See Figure 1.22.

(a) Find $N(1)$ and $N(2)$.
(b) Do you think it is true for all t that

$$N(t+1) = N(t) + 1?$$

If so, give an argument. If not, give a counterexample.
(c) Answer the same question as part (b), but with

$$N(t+1) = N(t) + N(1).$$

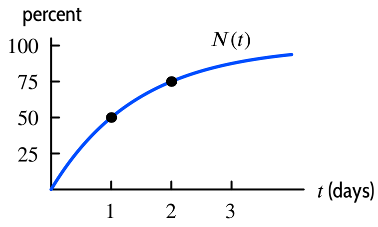

Figure 1.22

66. An alternative to petroleum-based diesel fuel, biodiesel, is derived from renewable resources such as food crops, algae, and animal oils. The table shows the recent annual percent growth in US biodiesel exports.[13]

(a) Find the largest time interval over which the percentage growth in the US exports of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.
(b) Find the largest time interval over which the actual US exports of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

Year	2012	2013	2014	2015	2016	2017	2018
% growth over previous yr	69.9	53.0	−57.8	5.93	0.33	6.20	10.10

67. Hydroelectric power is electric power generated by the force of moving water. Figure 1.23 shows[14] the annual percent growth in hydroelectric power consumption by the US industrial sector between 2008 and 2018.

(a) Find the largest time interval over which the percentage growth in the US consumption of hydroelectric power was an increasing function of time. Interpret what increasing means, practically speaking, in this case.
(b) Find the largest time interval over which the actual US consumption of hydroelectric power was a decreasing function of time. Interpret what decreasing means, practically speaking, in this case.

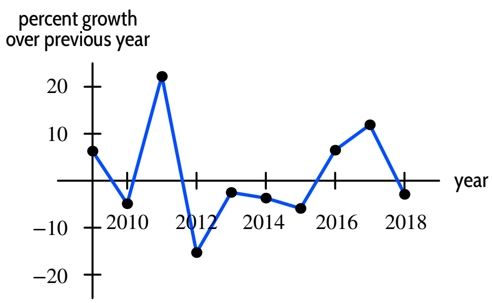

Figure 1.23

[13] www.eia.doe.gov, accessed August 19, 2019.
[14] Yearly values have been joined with line segments to highlight trends in the data; however, values in between years should not be inferred from the segments. From www.eia.doe.gov, accessed August 19, 2019.

68. Solar panels are arrays of photovoltaic cells that convert solar radiation into electricity. The table shows the annual percent change in the US price per watt of a solar panel.[15]

(a) Find the largest time interval over which the percentage growth in the US price per watt of a solar panel was a decreasing function of time. Interpret what decreasing means, practically speaking, in this case.

(b) Find the largest time interval over which the actual price per watt of a solar panel was a decreasing function of time. Interpret what decreasing means, practically speaking, in this case.

Year	2005	2006	2007	2008	2009	2010
% growth over previous yr	6.7	9.7	−3.7	3.6	−20.1	−29.7

69. Table 1.4 shows the average annual sea level, S, in meters, in Aberdeen, Scotland,[16] as a function of time, t, measured in years before 2014.

Table 1.4

t	0	25	50	75	100	125
S	7.071	7.083	6.990	6.964	6.985	6.900

(a) What was the average sea level in Aberdeen in 2014?

(b) In what year was the average sea level 7.083 meters? 6.985 meters?

(c) Table 1.5 gives the average sea level, S, in Aberdeen as a function of the year, x. Complete the missing values.

Table 1.5

x	1889	?	1939	1964	1989	2014
S	?	6.985	?	6.990	?	?

70. The table gives the required standard weight, w, in kilograms, of American soldiers, aged between 21 and 27, for height, h, in centimeters.[17]

(a) How do you know that the data in this table could represent a linear function?

(b) Find weight, w, as a linear function of height, h. What is the slope of the line? What are the units for the slope?

(c) Find height, h, as a linear function of weight, w. What is the slope of the line? What are the units for the slope?

h (cm)	172	176	180	184	188	192	196
w (kg)	79.7	82.4	85.1	87.8	90.5	93.2	95.9

71. Table 1.6 shows the pressure P, in torr,[18] at a depth of h meters below the surface of a lake.

(a) Explain why P could be a linear function of h.

(b) Find pressure, P, as a linear function of height h and give practical interpretations of the slope and vertical intercept.

(c) Use part (b) to approximate the depth at which the pressure is twice that at the surface of the lake.

Table 1.6

h (m)	6	8	10	12	14	16
P (torr)	1201	1348	1495	1642	1789	1936

72. A \$25,000 vehicle depreciates \$2000 a year as it ages. Repair costs are \$1500 per year.

(a) Write formulas for each of the two linear functions at time t, value, $V(t)$, and repair costs to date, $C(t)$. Graph them.

(b) One strategy is to replace a vehicle when the total cost of repairs is equal to the current value. Find this time.

(c) Another strategy is to replace the vehicle when the value of the vehicle is some percent of the original value. Find the time when the value is 6%.

73. A bakery owner knows that customers buy a total of q cakes when the price, p, is no more than $p = d(q) = 20 - q/20$ dollars. She is willing to make and supply as many as q cakes at a price of $p = s(q) = 11 + q/40$ dollars each. (The graphs of the functions $d(q)$ and $s(q)$ are called a *demand curve* and a *supply curve*, respectively.) The graphs of $d(q)$ and $s(q)$ are in Figure 1.24.

(a) Why, in terms of the context, is the slope of $d(q)$ negative and the slope of $s(q)$ positive?

(b) Is each of the ordered pairs (q, p) a solution to the inequality $p \le 20 - q/20$? Interpret your answers in terms of the context.

$$(60, 18) \qquad (120, 12)$$

(c) Graph in the qp-plane the solution set of the system of inequalities $p \le 20 - q/20$, $p \ge 11 + q/40$. What does this solution set represent in terms of the context?

(d) What is the rightmost point of the solution set you graphed in part (c)? Interpret your answer in terms of the context.

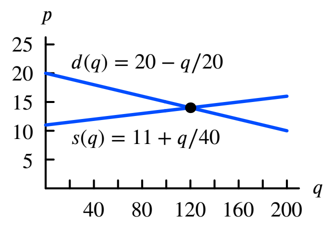

Figure 1.24

[15] We use the official price per peak watt, which uses the maximum number of watts a solar panel can produce under ideal conditions. From www.eia.doe.gov, accessed March 29, 2015.

[16] www.psmsl.org, accessed August 19, 2019.

[17] Adapted from usmilitary.about.com, accessed March 29, 2015.

[18] A torr is a unit of pressure.

74. **(a)** Consider the functions graphed in Figure 1.25(a). Find the coordinates of C.
(b) Consider the functions in Figure 1.25(b). Find the coordinates of C in terms of b.

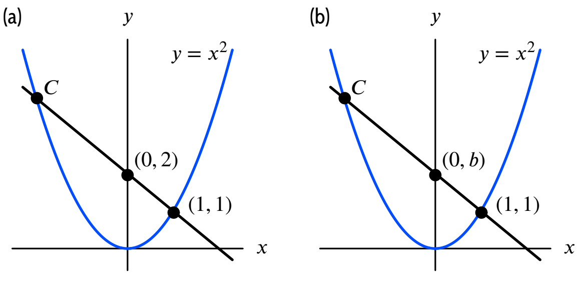

Figure 1.25

75. When Galileo was formulating the laws of motion, he considered the motion of a body starting from rest and falling under gravity. He originally thought that the velocity of such a falling body was proportional to the distance it had fallen.

What do the experimental data in Table 1.7 tell you about Galileo's hypothesis? What alternative hypothesis is suggested by the two sets of data in Table 1.7 and Table 1.8?

Table 1.7

Distance (ft)	0	1	2	3	4
Velocity (ft/sec)	0	8	11.3	13.9	16

Table 1.8

Time (sec)	0	1	2	3	4
Velocity (ft/sec)	0	32	64	96	128

Strengthen Your Understanding

■ In Problems 76–80, explain what is wrong with the statement.

76. For constants m and b, the slope of the linear function $y = b + mx$ is $m = \Delta x/\Delta y$.

77. The lines $x = 3$ and $y = 3$ are both linear functions of x.

78. The line $y - 3 = 0$ has slope 1 in the xy-plane.

79. Values of y on the graph of $y = 0.5x - 3$ increase more slowly than values of y on the graph of $y = 0.5 - 3x$.

80. The equation $y = 2x + 1$ indicates that y is directly proportional to x with a constant of proportionality 2.

■ In Problems 81–82, give an example of:

81. A linear function with a positive slope and a negative x-intercept.

82. A formula representing the statement "q is inversely proportional to the cube root of p and has a positive constant of proportionality."

■ In Problems 83–88, is the statement true or false? Give an explanation for your answer.

83. For any two points in the plane, there is a linear function whose graph passes through them.

84. If $y = f(x)$ is a linear function, then increasing x by 1 unit changes the corresponding y by m units, where m is the slope.

85. The linear functions $y = -x + 1$ and $x = -y + 1$ have the same graph.

86. The linear functions $y = 2 - 2x$ and $x = 2 - 2y$ have the same graph.

87. If y is a linear function of x, then the ratio y/x is constant for all points on the graph at which $x \neq 0$.

88. If $y = f(x)$ is a linear function, then increasing x by 2 units adds $m + 2$ units to the corresponding y, where m is the slope.

89. Which of the following functions has its domain identical with its range?

(a) $f(x) = x^2$ **(b)** $g(x) = \sqrt{x}$
(c) $h(x) = x^3$ **(d)** $i(x) = |x|$

1.2 EXPONENTIAL FUNCTIONS

Population Growth

The population of Burkina Faso, a sub-Saharan African country,[19] is given in Table 1.9. To see how the population is growing, we look at the increase in population in the third column. If the population had been growing linearly, all the numbers in the third column would be the same.

[19] www.worldometers.inf, accessed August 19, 2019.

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